Title of Unit	The Number System	**Grade Level**	8th grade Level 1 Supplemental
Curriculum Area	Mathematics	**Time Frame**	3-4 weeks
Developed By	Munira Jamali		
Identify Desired Results (Stage 1)			
Content Standards			

Number System

6.NS.1.

Interpret and compute quotients of fractions, and solve word problems involving division of fractions by fractions, e.g., by using visual fraction models and equations to represent the problem. For example, create a story context for (2/3) ÷ (3/4) and use a visual fraction model to show the quotient; use the relationship between multiplication and division to explain that (2/3) ÷ (3/4) = 8/9 because 3/4 of 8/9 is 2/3. (In general, (a/b) ÷ (c/d) = ad/bc.)

How much chocolate will each person get if 3 people share 1/2 lb of chocolate equally? How many 3/4-cup servings are in 2/3 of a cup of yogurt? How wide is a rectangular strip of land with length 3/4 mi and area 1/2 square mi?

6.NS.2.

Fluently divide multi-digit numbers using the standard algorithm. 6.NS.3. Fluently add, subtract, multiply, and divide multi-digit decimals using the standard algorithm for each operation. 6.NS.4. Find the greatest common factor of two whole numbers less than or equal to 100 and the least common multiple of two whole numbers less than or equal to 12. Use the distributive property to express a sum of two whole numbers 1–100 with a common factor as a multiple of a sum of two whole numbers with no common factor. For example, express 36 + 8 as 4 (9 + 2).

6.NS.5.

Understand that positive and negative numbers are used together to describe quantities having opposite directions or values (e.g., temperature above/below zero, elevation above/below sea level, credits/debits, positive/negative electric charge); use positive and negative numbers to represent quantities in real-world contexts, explaining the meaning of 0 in each situation.

6.NS.6.

Understand a rational number as a point on the number line. Extend number line diagrams and coordinate axes familiar from previous grades to represent points on the line and in the plane with negative number coordinates.

a. Recognize opposite signs of numbers as indicating locations on opposite sides of 0 on the number line; recognize that the opposite of the opposite of a number is the number itself, e.g.,–(–3) = 3, and that 0 is its own opposite.

b. Understand signs of numbers in ordered pairs as indicating locations in quadrants of the coordinate plane; recognize that when two ordered pairs differ only by signs, the locations of the points are related by reflections across one or both axes. c. Find and position integers and other rational numbers on a horizontal or vertical number line diagram; find and position pairs of integers and other rational numbers on a coordinate plane.

6.NS.7.

Understand ordering and absolute value of rational numbers. a. Interpret statements of inequality as statements about the relative position of two numbers on a number line diagram. For example, interpret –3 > –7 as a statement that –3 is located to the right of –7 on a number line oriented from left to right. b. Write, interpret, and explain statements of order for rational numbers in real-world contexts. For example, write –3°C > –7°C to express the fact that –3°C is warmer than –7°C. c. Understand the absolute value of a rational number as its distance from 0 on the number line; interpret absolute value as magnitude for a positive or negative quantity in a real-world situation. For example, for an account balance of –30 dollars, write |–30| = 30 to describe the size of the debt in dollars. d. Distinguish comparisons of absolute value from statements about order. For example, recognize that an account balance less than –30 dollars represents a debt greater than 30 dollars. 6.NS.8. Solve real-world and mathematical problems by graphing points in all four quadrants of the coordinate plane. Include use of coordinates and absolute value to find distances between points with the same first coordinate or the same second coordinate.

Understandings	Essential Questions	
Overarching Understanding	**Overarching**	**Topical**
Procedures used for dividing fractions can be logically explained in several ways. The system of rational numbers includes negative numbers as well as positive ones. • Rational number can be arranged in order. • Absolute value can be described in more than way, depending upon the real-world context. It can be distance, or it can be size (magnitude). • Points can be graphed in all four quadrants of a coordinate grid by using ordered pairs to determine location. • A rational number can be represented as a point on a number line and the number line can be used as a tool to order rational numbers.	How can you compute fractions by using visual fraction models and equations? 2. How do you find the GCF of two whole numbers using the distributive property? 3. How do you use positive and negative numbers to describe quantities having opposite values? 4. What is a rational number and how can you graph it? 5. What is absolute value? 6. How can you apply inverse operations in solving problem	How do number properties assist in computation? Is estimation more appropriate than finding an exact answer? How do we use ordinal numbers in everyday life?
DIFFERENTIATION		

UDL/ FRONT LOADING
Students apply and extend their understanding of number sense, computation with multi-digit whole numbers and decimals (to hundredths), including application of order of operations, addition, subtraction, multiplication, and division of common fractions, and familiarity with factors and multiples.
Front load vocabulary associated with applications of integers such as:
Thermometer
Elevator Credit/Debit
Sea level

ACCELERATION
Have students describe quantities having opposite directions or values (e.g., temperature above/below zero, elevation above/below sea level, credits/debits, positive/negative electric charge.
Students design a story problems using temperature above/below zero, elevation above/below sea level, credits/debits, positive/negative electric charge.
Explain absolute value by using the distance they travel to school each way (to and fro). That distance is always positive.
Provide a scenario where students will gather real – world data and graphing points in all four quadrants of the coordinate plane. Include use of coordinates and absolute value to find distances between points with the same first coordinate or the same second coordinate

INTERVENTION
Intervention for low achieving students and students with disabilities:
• Small teacher to student ratio discussion • Emphasize think-pair-share • Make connections to real life • Give concrete examples • Use of manipulatives – especially the number line • Use of multiple representations to represent fraction division problems. Set the problem in context and represent the problem with a concrete or pictorial model. • Provide multiple experiences to understand the relationships between numbers, absolute value, and statements about order
Example: in real world, the absolute value can be used to describe size or magnitude. An ocean depth of 900 feet, write | –900| = 900 to describe the distance below sea level

Knowledge Students will know…	**Skills** Students will be able to…

Description of the critical area: Students use the meaning of fractions, the meanings of multiplication and division, and the relationship between multiplication and division to understand and explain why the procedures for dividing fractions make sense. Students use these operations to solve problems. Students extend their previous understandings of number and the ordering of numbers to the full system of rational numbers, which includes negative rational numbers, and in particular negative integers. They reason about the order and absolute value of rational numbers and about the location of points in all four quadrants of the coordinate plan	Apply and extend previous understandings of multiplication and division to divide fractions by fractions. Compute fluently with multi-digit numbers and find common factors and multiples. Apply and extend previous understandings of numbers to the system of rational numbers.

Assessment Evidence (Stage 2)

Performance Task Description

• **Goal** • **Role** • **Audience** • **Situation** • **Product/ Performance** • **Standards**	**6.NS.4 Pedro's Tables** http://paineinthemath.net/wp-content/uploads/2014/06/MARS-Grade-7-Pedros-Tables.pdf This problem gives you the chance to: • work with number properties including divisibility • explain your reasoning Standards 6.NS.4. Find the greatest common factor of two whole numbers less than or equal to 100 and the least common multiple of two whole numbers less than or equal to 12. Use the distributive property to express a sum of two whole numbers 1–100 with a common factor as a multiple of a sum of two whole numbers with no common factor. For example, express 36 + 8 as 4 (9 + 2). **6.NS Interpreting Multiplication and Division** http://map.mathshell.org/lessons.php?unit=6115&collection=8&redir=1 This lesson unit is designed to help students to interpret the meaning of multiplication and division. Many students have a very limited understanding of these operations and only recognise them in terms of 'times' and 'share'. They find it hard to give any meaning to calculations that involve nonintegers. This is one reason why they have difficulty when choosing the correct operation to perform when solving word problem. 6.NS: Apply and extend previous understandings of multiplication and division to divide fractions by fractions **Adding and Subtracting Directed Numbers** http://map.mathshell.org/materials/download.php?fileid=1556 This lesson unit is intended to help students to: • Add and subtract directed numbers (positive, negative and zero) with understanding. • Address common misconceptions about the addition and subtraction of directed numbers. • Explain their reasoning using diagrams. Standards 6.NS: Apply and extend previous understandings of numbers to the system of rational number
Other Evidence	

Illustrative Mathematics

• 6.NS Cup of Rice •
6.NS Dan's Division Strategy

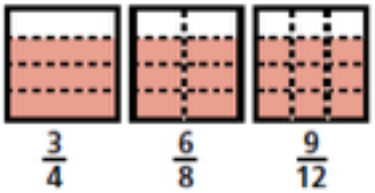
• 6.NS Interpreting a Division Computation

Learning Plan (Stage 3)

- **Where are your students headed? Where have they been? How will you make sure the students know where they are going?**
- **How will you hook students at the beginning of the unit?**
- **What events will help students experience and explore the big idea and questions in the unit? How will you equip them with needed skills and knowledge?**
- **How will you cause students to reflect and rethink? How will you guide them in rehearsing, revising, and refining their work?**
- **How will you help students to exhibit and self-evaluate their growing skills, knowledge, and understanding throughout the unit?**
- **How will you tailor and otherwise personalize the learning plan to optimize the engagement and effectiveness of ALL students, without compromising the goals of the unit?**
- **How will you organize and sequence the learning activities to optimize the engagement and achievement of ALL students?**

Use of number line
Use of human graph
Using common denominators to divide fractions Journal / Quick Write Prompts
Use of visual fraction models for division
Using common denominators to divide fractions to understand the remainder
Sorting cards
Fraction bars in teaching equivalent fractions
Vocabulary Development – 3x3 EL puzzle

Equivalent fractions are fractions that represent the same part of a whole.
• Students can fold paper to investigate equivalent fractions
Show This:



Ask: What happened to the fourths when the paper was folded? What happened to the number of shaded parts when the paper was folded? What equivalent fraction could represent the shaded sections now? Is the same part of the whole shaded? • Show symbolically how one fraction is based on the other.

$$\frac{3 \times 2}{4 \times 2} = \frac{6}{8} \quad \text{and} \quad \frac{6 \div 2}{8 \div 2} = \frac{3}{4}$$

Help students generalize that the numerator and denominator can both be multiplied or divided by the same number to obtain an equivalent fraction
Vocabulary
Development Simplest form is sometimes referred to as lowest terms. Students should become familiar with both expressions. The term reducing has often been used to describe the process of obtaining simplest form. However, this term can be misleading since the fractions are not actually not reduced in size
Mixed Numbers
Fraction models and other area models are useful in illustrating mixed numbers.

Show This:

Title of Unit	**Developing Understanding and Application of Proportional Relationships**				
Curriculum Area	Mathematics	**Time Frame**	3-4 weeks		
Developed By	Munira Jamali				
Identify Desired Results (Stage 1)					
Content Standards					

7.RP.1
Compute unit rates associated with ratios of fractions, including ratios of lengths, areas and other quantities measured in like or different units. For example, if a person walks 1/2 mile in each 1/4 hour, compute the unit rate as the complex fraction 1/2/1/4 miles per hour, equivalently 2 miles per hour.

7.RP.2
Recognize and represent proportional relationships between quantities. a. Decide whether two quantities are in a proportional relationship, e.g., by testing for equivalent ratios in a table or graphing on a coordinate plane and observing whether the graph is a straight line through the origin. b. Identify the constant of proportionality (unit rate) in tables, graphs, equations, diagrams, and verbal descriptions of proportional relationships. c. Represent proportional relationships by equations. For example, if total cost t is proportional to the number n of items purchased at a constant price p, the relationship between the total cost and the number of items can be expressed as t = pn. d. Explain what a point (x, y) on the graph of a proportional relationship means in terms of the situation, with special attention to the points (0, 0) and (1, r) where r is the unit rate.

7.RP.3
Use proportional relationships to solve multistep ratio and percent problems. Examples: simple interest, tax, markups and markdowns, gratuities and commissions, fees, percent increase and decrease, percent error.

Understandings	Essential Questions	
Overarching Understanding	**Overarching**	**Topical**

Proportional reasoning is essential in problem solving • Understanding mathematical relationships allows us to make predictions, calculate and model unknown quantities. • Proportional relationships express how quantities change in relationship to each other. (Look at NYC Common Core Grade 7)	How can proportions be used to solve problems? • When is a relationship proportional? • How can proportions increase our understanding of the real world? • How does the mathematical use of the word similar differ from the everyday use? • How can similarity help us solve measurement	Why are ratios and proportions important?? Does my answer make sense? How can I compare two different things? (How can we show the relationship between two quantities or values?) How should I solve it? (What number form should be used to solve a problem?)	
Related Misconceptions			

Lessons may need to be customized if the class period is not long enough to do all of what is presented and/or if students lack prerequisite skills and understanding to move through the entire lesson in the time allotted. A suggestion for customizing the lesson is to first decide upon and designate each question, example, exercise, or challenge as either "Must Do" or "Could Do." A: Select "Must Do" dialogue, questions, and problems that meet the Student Outcome(s) while still providing a coherent experience for students; reference the ladder. The expectation should be that the majority of the class will be able to complete the "Must Do" portions of the lesson within the allocated time. While choosing the "Must Do" portions of the lesson, keep in mind the need for a balance of dialogue and conceptual questioning, application problems, and abstract problems, and a balance between students using pictorial/ graphical representations and abstract representations. Highlight dialogue to be included in the delivery of instruction	measurement problems? • What are the connections between similarity, geometry and algebra	

Knowledge Students will know…	**Skills** Students will be able to…

To extend their understanding of ratios and develop understanding of proportionality to solve single- and multi-step problems. To use their understanding of ratios and proportionality to solve a wide variety of percent problems, including those involving discounts, interest, taxes, tips, and percent increase or decrease. To solve problems about scale drawings by relating corresponding lengths between the objects or by using the fact that relationships of lengths within an object are preserved in similar objects. To graph proportional relationships and understand the unit rate informally as a measure of the steepness of the related line, called the slope. To distinguish proportional relationships from other relationships To reason about relationships among two-dimensional figures using scale drawings and informal geometric constructions, and they gain familiarity with relationships between angles formed by Intersecting lines.	Analyze proportional relationships and use them to solve real-world and mathematical problem Draw, construct, and describe geometrical figures and describe the relationships between them Make sense of problems and persevere in solving them. Reason abstractly and quantitatively To Draw, construct, and describe geometrical figures and describe the relationships between them.

Assessment Evidence (Stage 2)

Performance Task Description

- **Goal**
- **Role**
- **Audience**
- **Situation**
- **Product/ Performance**
- **Standards**

Proportional **Relationship**

Focus Standard:
7.RP.A.2 Recognize and represent proportional relationships between quantities.
a. Decide whether two quantities are in a proportional relationship, e.g., by testing for equivalent ratios in a table or graphing on a coordinate plane and observing whether the graph is a straight line through the origin.

Lesson 1: An Experience in Relationships as Measuring Rate (P)1
Lesson 2: Proportional Relationships (P)
Lessons 3–4: Identifying Proportional and Non-Proportional Relationships in Tables (P, P)
Lessons 5–6: Identifying Proportional and Non-Proportional Relationships in Graphs (E, E)
Goals
In Lesson 1 of Topic A, students are reintroduced to the meanings of value of a ratio, equivalent ratios, rate, and unit rate through a collaborative work task where they record their rates choosing an appropriate unit of rate measurement.
In Lesson 2, students conceptualize that two quantities are proportional to each other when there exists a constant such that each measure in the first quantity multiplied by this constant gives the corresponding measure in the second quantity (7.RP.A.2). They then apply this basic understanding in Lessons 3–6 by examining situations to decide whether two quantities are in a proportional or non-proportional relationship by first checking for a constant multiple between measures of the two quantities, when given a table, and then by graphing on a coordinate plane. Students recognize that the graph of a proportional relationship must be a straight line through the origin (7.RP.A.2

Other Evidence

Assessment Type	Administered	Format	Standards Addressed
Mid-Module Assessment Task	after topic A	constructed response with rubric	7.RP.A.2
End of Module Assessment Task	After Topic D	Constructed response with rubric	7.RP.A.1, 7.RP.A.2, 7.RP.A.3, 7.EE.B.4a, 7.G.A.1

Learning Plan (Stage 3)

• **Where are your students headed? Where have they been? How will you make sure the students know where they are going?** • **How will you hook students at the beginning of the unit?** • **What events will help students experience and explore the big idea and questions in the unit? How**	Engage NY Module1(Ratios and Proportion) Preparation of lessons will be more effective and efficient if there has been an adequate analysis of the module first. Each module in A Story of Ratios can be compared to a chapter in a book. How is the module moving the plot, the mathematics, forward? What new learning is taking place? How are the topics and objectives building on one another? The following is a suggested process for preparing to teach a module. Step 1: Get a preview of the plot. A: Read the Table of Contents. At a high level, what is the plot of the module? How does the story develop across the topics? B: Preview the module's Exit Tickets to see the trajectory of the module's mathematics and the nature of the work students are expected to be able to do. Note: When studying a PDF file, enter "Exit Ticket" into the search feature to navigate from one Exit Ticket to the next. Step 2: Dig into the details. A: Dig into a careful reading of the Module Overview. While reading the narrative, liberally reference the lessons and Topic Overviews to clarify the meaning of the text – the lessons demonstrate the strategies, show how to use the models, clarify vocabulary, and build understanding of concepts. B: Having thoroughly investigated the Module Overview, read through the Student Outcomes of each lesson (in

Title of Unit	Understand Expressions and Equations	**Grade Level**	Level 1 Supplemental
Curriculum Area	Mathematics	**Time Frame**	3-5 weeks
Developed By	Munira Jamali		
Identify Desired Results (Stage 1)			
Content Standards			

7.EE.1.
Apply properties of operations as strategies to add, subtract, factor, and expand linear expressions with rational coefficients

7.EE.2.
Understand that rewriting an expression in different forms in a problem context can shed light on the problem and how the quantities in it are related. For example, a + 0.05a = 1.05a means that "increase by 5%" is the same as "multiply by 1.05."

7.EE.3.
Solve multi-step real-life and mathematical problems posed with positive and negative rational numbers in any form (whole numbers, fractions, and decimals), using tools strategically. Apply properties of operations to calculate with numbers in any form; convert between forms as appropriate; and assess the reasonableness of answers using mental computation and estimation strategies. For example: If a woman making $25 an hour gets a 10% raise, she will make an additional 1/10 of her salary an hour, or $2.50, for a new salary of $27.50. If you want to place a towel bar 9 3/4 inches long in the center of a door that is 27 1/2 inches wide, you will need to place the bar about 9 inches from each edge; this estimate can be used as a check on the exact computation.

7.EE.4.
Use variables to represent quantities in a real-world or mathematical problem, and construct simple equations and inequalities to solve problems by reasoning about the quantities. a. Solve word problems leading to equations of the form px + q = r and p(x + q) = r, where p, q, and r are specific rational numbers. Solve equations of these forms fluently. Compare an algebraic solution to an arithmetic solution, identifying the sequence of the operations used in each approach. For example, the perimeter of a rectangle is 54 cm. Its length is 6 cm. What is its width? b. Solve word problems leading to inequalities of the form px + q > r or px + q < r, where p, q, and r are specific rational numbers. Graph the solution set of the inequality and interpret it in the context of the problem. For example: As a salesperson, you are paid $50 per week plus $3 per sale. This week you want your pay to be at least $100. Written inequality for the number of sales you need to make, and describe the solutions.

Understandings	Essential Questions	
Overarching Understanding	**Overarching**	**Topical**
Generating equivalent, linear expressions with rational coefficients using the properties of operations will lead to solving linear equation. • Discovering that rewriting expressions in different forms in a problem context leads to understanding that the values are equivalent. • Ability to solve and explain real life and mathematical problems involving rational numbers using numerical and algebraic expressions is important for preparation for HS Algebra. • Constructing simple equations and inequalities to solve real life word problems is a necessary concept.	How can I apply the order of operations and the fundamentals of algebra to solve problems? How can I justify that multiple representations in the context of a problem are equivalent expressions? How do I assess the reasonableness of my answer? How will I use the	Do mathematical models conceal as much as they reveal? What patterns or relationships do we see in each type of mathematics? What are the different ways to represent the patterns or relationships?
Related Misconceptions		

As students begin to build and work with expressions containing more than two operations, students tend to set aside the order of operations. For example having a student simplify an expression like 8 + 4(2x - 5) + 3x can bring to light several misconceptions. Do the students immediately add the 8 and 4 before distributing the 4? Do they only multiply the 4 and the 2x and not distribute the 4 to both terms in the parenthesis? Do they collect all like terms 8 + 4 – 5, and 2x + 3x? Each of these show gaps in students' understanding of how to simplify numerical expressions with multiple operations.	properties of equality to explain the order of the steps in solving equations and inequalities? How do I interpret the solutions for equations and inequalities in the context of the problem?	What different interpretations can be obtained from a particular pattern or relationship? What predictions can the patterns or relationships support? How can we use or test our predictions? Are they valid? Are they significant? Where in the real world would I find patterns?

Knowledge Students will know…	**Skills** Students will be able to…
To understand equivalent expressions as they apply the properties of operations (associative, commutative, and distributive) to write expressions in both standard form (by expanding products into sums) and in factored form (by expanding sums into products). To use linear equations to solve unknown angle problems and other problems presented within context to understand that solving algebraic equations is all about the numbers. It is assumed that a number already exists to satisfy the equation and context; we just need to discover it. A number sentence is an equation that is said to be true if both numerical expressions evaluate to the same number; it is said to be false otherwise. To use the number line to understand the properties of inequality and recognize when to preserve the inequality and when to reverse the inequality when solving problems leading to inequalities.	• Apply properties of operations to generate equivalent expressions • Determine solutions to real-life and mathematical problems by using numerical and algebraic expressions and equations • Compute solutions to real-life and mathematical problems involving angle measure, area, surface area, and volume • Apply properties of operations to generate equivalent expressions • Solve real-life and mathematical problems using numerical and algebraic expressions and equations

Assessment Evidence (Stage 2)

Performance Task Description

• **Goal** • **Role** • **Audience** • **Situation** • **Product/Performance** • **Standards**	Expressions and Equations NY module 3 **Topic A:** Use Properties of Operations to Generate Equivalent Expressions (7.EE.A.1, 7.EE.A.2) Lessons 1–2: Generating Equivalent Expressions Lessons 3–4: Writing Products as Sums and Sums as Products Lesson 5: Using the Identity and Inverse to Write Equivalent Expressions. Lesson 6: Collecting Rational Number Like Terms **Topic B:** Solve Problems Using Expressions, Equations, and Inequalities (7.EE.B.3, 7.EE.B.4, 7.G.B.5) Lesson 7: Understanding Equations Lessons 8–9: Using If-Then Moves in Solving Equations Lessons 10–11: Angle Problems and Solving Equations Lesson 12: Properties of Inequalities Lesson 13: Inequalities Lesson 14: Solving Inequalities Lesson 15: Graphing Solutions to Inequalities.

Other Evidence

Assessment Type	Administered	Format	Standards Addressed
Mid-Module Assessment Task	After Topic B	Constructed response with rubric	7.EE.A.1, 7.EE.A.2, 7.EE.B.3, 7.EE.B.4, 7.G.B.5
End-of-Module Assessment Task	After Topic C	Constructed response with rubric	7.EE.A.1, 7.EE.A.2, 7.G.B.4, 7.G.B.5, 7.G.B.6

Learning Plan (Stage 3)

• **Where are your students headed? Where have they been? How will you make sure the students know where they are going?** • **How will you hook students at the beginning of the unit?** • **What events will help students experience and explore the big idea and questions in the unit? How will you equip them with needed skills and knowledge?** • **How will you cause students to reflect and rethink? How will you guide them in rehearsing, revising, and refining their work?** • **How will you help students to exhibit and self-evaluate their growing skills, knowledge, and understanding throughout the unit?** • **How will you tailor and otherwise personalize the learning plan to optimize the engagement and effectiveness of ALL students, without compromising the goals of the unit?** • **How will you organize and sequence the learning activities to optimize the engagement and achievement of ALL students?**	**Topic A:** **Use Properties of Operations to Generate Equivalent Expressions** To begin this module, students will generate equivalent expressions using the fact that addition and multiplication can be done in any order with any grouping and will extend this understanding to subtraction (adding the inverse) and division (multiplying by the multiplicative inverse, also known as the reciprocal) (7.EE.A.1). They extend the properties of operations with numbers (learned in earlier grades) and recognize how the same properties hold true for letters that represent numbers. Knowledge of rational number operations from Module 2 is demonstrated as students collect like terms containing both positive and negative integers. An area model is used as a tool for students to rewrite products as sums and sums as products and to provide a visual representation leading students to recognize the repeated use of the distributive property in factoring and expanding linear expressions (7.EE.A.1). Students examine situations where more than one form of an expression may be used to represent the same context, and they see how looking at each form can bring a new perspective (and thus deeper understanding) to the problem. Students recognize and use the identity properties and the existence of additive inverses to efficiently write equivalent expressions in standard form, for example, $2xx$ + $(-2xx)$ + 3 = 0 + 3 = 3 (7.EE.A.2). By the end of the topic, students have the opportunity to practice knowledge of operations with rational numbers gained in Module 2 (7.NS.A.1, 7.NS.A.2) as they collect like terms with rational number coefficients (7.EE.A.1). **Topic B:** **Solve Problems Using Expressions, Equations, and Inequalities** students use linear equations and inequalities to solve problems (7.EE.B.4). They continue to use tape diagrams from earlier grades where they see fit, but will quickly discover that some problems would more reasonably be solved algebraically (as in the case of large numbers). Guiding students to arrive at this realization on their own develops the need for algebra. This algebraic approach builds upon work in Grade 6 with equations (6.EE.B.6, 6.EE.B.7) to now include

Title of Unit	Understanding Geometry	Grade Level	Grade 7 Supplemental
Curriculum Area	Mathematics	Time Frame	5-6 weks
Developed By	Munira Jamali		

Identify Desired Results (Stage 1)

Content Standards

7.G.1. Solve problems involving scale drawings of geometric figures, including computing actual lengths and areas from a scale drawing and reproducing a scale drawing at a different scale.
7.G.2. Draw (freehand, with ruler and protractor, and with technology) geometric shapes with given conditions. Focus on constructing triangles from three measures of angles or sides, noticing when the conditions determine a unique triangle, more than one triangle, or no triangle.
7.G.3. Describe the two-dimensional figures that result from slicing three-dimensional figures, as in plane sections of right rectangular prisms and right rectangular pyramids.
7.G.3.1 Describe how two or more objects are related in space (e.g., skew lines, the possible ways three planes might intersect).CA (s/a)2 Solve real-life and mathematical problems involving angle measure, area, surface area, and volume.
7.G.4. Know the formulas for the area and circumference of a circle and use them to solve problems; give an informal derivation of the relationship between the circumference and area of a circle
7.G.5. Use facts about supplementary, complementary, vertical, and adjacent angles in a multistep problem to write and solve simple equations for an unknown angle in a figure.
7.G.6. Solve real-world and mathematical problems involving area, volume and surface area of two- and three dimensional objects composed of triangles, quadrilaterals, polygons, cubes, and right prisms.

Understandings	Essential Questions	
Overarching Understanding	**Overarching**	**Topical**
Students delve further into several geometry topics they have been developing over the years. Grade 7 presents some of these topics, (e.g., angles, area, surface area, and volume) in the most challenging form students have experienced yet. This unit assumes students understand the basics. The goal is to build a fluency in these difficult problems. The remaining topics, (i.e., working on constructing triangles and taking slices (or cross-sections) of three-dimensional figures) are new to students.	What 2-D figure results from slicing 3-D figures? (cones, spheres, or cylinders) How do you find the surface area and volume of a 3D figure? What is the total number of degrees in supplementary and complementary angles? What is the relationship between vertical and adjacent angles? How would the volume and surface area be affected when dimensions of a figure are doubled and/or triple?	Where in the real world can I find shapes? Where would you find symmetry? How can objects be represented and compared using geometric attributes? Is geometry more like map-making and using a map, or inventing and
Related Misconceptions		

Student's may have misconceptions about correctly setting up proportions, how to read a ruler, doubling side measures, and does not double perimeter. Students may believe: Pi is an exact number rather than understanding that 3.14 is just an approximation of pi. Many students are confused when dealing with circumference (linear measurement) and area. This confusion is about an attribute that is measured using linear units (surrounding) vs. an attribute that is measured using area units (covering).		inventing and playing games like chess? How can I identify and describe solid figures by describing the faces, edges, and sides?
Knowledge Students will know…	**Skills** Students will be able to…	
To continue their work with area from Grade 6, solving problems Involving the area and circumference of a circle and surface area of three-dimensional objects. To reason about relationships among two-dimensional figures using scale drawings and informal geometric constructions, and they gain familiarity with relationships between angles formed by Intersecting lines. To work with three-dimensional figures, relating them to two-dimensional figures by examining cross-sections. To solve real-world and mathematical problems involving area, surface area, and volume of two- and three-dimensional objects composed of triangles, quadrilaterals, polygons, cubes, and right prisms.	Draw, construct, and describe geometrical figures and describe the relationships between them Solve real-life and mathematical problems involving angle measure, area, surface area, and volume.	
Assessment Evidence (Stage 2)		
Performance Task Description		

• **Goal** • **Role** • **Audience** • **Situation** • **Product/Performance** • **Standards**	**NY Engage Module 6** **Topic A Unknown Angles** Focus Standard: 7.G.B.5 Use facts about supplementary, complementary, vertical, and adjacent angles in a multi-step problem to write and solve simple equations for an unknown angle in a figure. Instructional Days: 4 Lesson 1: Complementary and Supplementary Angles Lessons 2–4: Solving for Unknown Angles Using Equations . **Topic B Constructing Triangles** Focus Standard: 7.G.A.2 Draw (freehand, with ruler and protractor, and with technology) geometric shapes with given conditions. Focus on constructing triangles from three measures of angles or sides, noticing when the conditions determine a unique triangle, more than one triangle, or no triangle. Instructional Days: 11 Lesson 5: Identical Triangles (S)1 Lesson 6: Drawing Geometric Shapes (E) Lesson 7: Drawing Parallelograms (P) Lesson 8: Drawing Triangles (E) Lesson 9: Conditions for a Unique Triangle—Three Sides and Two Sides and the Included Angle (E) Lesson 10: Conditions for a Unique Triangle—Two Angles and a Given Side (E) Lesson 11: Conditions on Measurements That Determine a Triangle (E) Lesson 12: Unique Triangles—Two Sides and a Non-Included Angle (E) Lessons 13–14: Checking for Identical Triangles (P, P) Lesson 15: Using Unique Triangles to Solve Real-World and Mathematical Problems (P) Topic C Slicing Solids Focus Standard: 7.G.A.3 Describe the two-dimensional figures that result from slicing three-dimensional figures, as in plane sections of right rectangular prisms and right rectangular pyramids. Instructional Days: 4 Lesson 16: Slicing a Right Rectangular Prism with a Plane (P)1 Lesson 17: Slicing a Right Rectangular Pyramid with a Plane (S) Lesson 18: Slicing on an Angle (P) Lesson 19: Understanding Three-Dimensional Figures (P)

Other Evidence

Mid-Module Assessment Task After Topic B Constructed response with rubric 7.G.B.2, 7.G.A.5
End-of-Module Assessment Task After Topic E Constructed response with rubric 7.G.A.2, 7.G.A.3, 7.G.B.5, 7.G.B.6

Learning Plan (Stage 3)

• **Where are your students headed? Where have they been? How will you make sure the students know where they are going?** • **How will you hook students at the beginning of the unit?** • **What events will help students experience and explore the big idea and questions in the unit? How will you equip them with needed skills and knowledge?** • **How will you cause students to reflect and rethink? How will you guide them in rehearsing, revising, and refining their work?** • **How will you help students to exhibit and self-evaluate their growing skills, knowledge, and understanding throughout the unit?** • **How will you tailor and otherwise personalize the learning plan to optimize the engagement and effectiveness of ALL students, without compromising the goals of the unit?** • **How will you organize and sequence the learning activities to optimize the engagement and achievement of ALL students?**	**In Topic A,** students solve for unknown angles. The supporting work for unknown angles began in Grade 4 Module 4 (4.MD.C.5, 4.MD.C.6, 4.MD.C.7), where all of the key terms in this topic were first defined, including the following: adjacent, vertical, complementary, and supplementary angles; angles on a line; and angles at a point. In Grade 4, students used those definitions as a basis to solve for unknown angles by using a combination of reasoning (through simple number sentences and equations) and measurement (using a protractor). For example, students learned to solve for an unknown angle in a pair of supplementary angles where one angle measurement is known. In Grade 7 Module 3, students studied how expressions and equations are an efficient way to solve problems. Two lessons were dedicated to applying the properties of equality to isolate the variable in the context of unknown angle problems. The diagrams in those lessons were drawn to scale to help students more easily make the connection between the variable and what it actually represents. Now in Module 6, the most challenging examples of unknown angle problems (both diagram-based and verbal) require students to use a synthesis of angle relationships and algebra. The problems are multi-step, requiring students to identify several layers of angle relationships and to fit them with an appropriate equation to solve. Unknown angle problems show students how to look for, and make use of, structure (MP.7). In this case, they use angle relationships to find the measurement of an angle. **Next, in Topic B,** students work extensively with a ruler, compass, and protractor to construct geometric shapes, mainly triangles (7.G.A.2). The use of a compass is new (e.g., how to hold it and how to create equal segment lengths). Students use the tools to build triangles with given conditions such as side length and the measurement of the included angle (MP.5). Students also explore how changes in arrangement and measurement affect a triangle, culminating in a list of conditions that determine a unique triangle. Students understand two new concepts about unique triangles. They learn that under a condition that determines a unique triangle: (1) a triangle can be drawn, and (2) any two triangles drawn under the condition are identical. It is important to note that there is no mention of congruence in the CCSS until Grade 8, after a study of rigid motions. Rather, the focus of Topic B is developing students' intuitive understanding of the structure of a triangle. This includes students noticing the conditions that

Title of Unit	Statistics and Probability	**Grade Level**	8th grade Level 1 Supplemental
Curriculum Area	Mathematics	**Time Frame**	5-6 weeks
Developed By	Munira Jamali		
Identify Desired Results (Stage 1)			
Content Standards			

7.SP.5. Understand that the probability of a chance event is a number between 0 and 1 that expresses the likelihood of the event occurring. Larger numbers indicate greater likelihood. A probability near 0 indicates an unlikely event, a probability around 1/2 indicates an event that is neither unlikely nor likely, and a probability near 1 indicates a likely event.
7.SP.6. Approximate the probability of a chance event by collecting data on the chance process that produces it and observing its long-run relative frequency, and predict the approximate relative frequency given the probability. For example, when rolling a number cube 600 times, predict that a 3 or 6 would be rolled roughly 200 times, but probably not exactly 200 times.
7.SP.7. Develop a probability model and use it to find probabilities of events. Compare probabilities from a model to observed frequencies; if the agreement is not good, explain possible sources of the discrepancy. a. Develop a uniform probability model by assigning equal probability to all outcomes, and use the model to determine probabilities of events. For example, if a student is selected at random from a class, find the probability that Jane will be selected and the probability that a girl will be selected. b. Develop a probability model (which may not be uniform) by observing frequencies in data generated from a chance process. For example, find the approximate probability that a spinning penny will land heads up or that a tossed paper cup will land open-end down. Do the outcomes for the spinning penny appear to be equally likely based on the observed frequencies?
7.SP.8. Find probabilities of compound events using organized lists, tables, tree diagrams, and simulation.
a. Understand that, just as with simple events, the probability of a compound event is the fraction of outcomes in the sample space for which the compound event occurs. b. Represent sample spaces for compound events using methods such as organized lists, tables and tree diagrams. For an event described in everyday language (e.g., "rolling double sixes"), identify the outcomes in the sample space which compose the event. c. Design and use a simulation to generate frequencies for compound events. For example, use random digits as a simulation tool to approximate the answer to the question: If 40% of donors have type A blood, what is the probability that it will take at least 4 donors to find one with type A blood?

Understandings	Essential Questions	
Overarching Understanding	**Overarching**	**Topical**
In this unit students begin their study of probability, learning how to interpret probabilities and how to compute probabilities in simple settings. They also learn how to estimate probabilities empirically. Probability provides a foundation for the inferential reasoning developed in the second half of this module. Students build on their knowledge of data distributions that they studied in Grade 6, compare data distributions of two or more populations, and are introduced to the idea of drawing informal inferences based on data from random samples	Probability and Statistics How do you determine which measures of variability should be used to draw informal comparative inferences? How are lists, tables, tree diagrams or simulation used to find the probability of an event? How is probability used to predict frequency of an event?	
Related Misconceptions		

Students may believe: One random sample is not representative of the entire population. Many samples must be taken in order to make an inference that is valid. By comparing the results of one random sample with the results of multiple random samples, students can correct this misconception Students often expect the theoretical and experimental probabilities of the same data to match. By providing multiple opportunities for students to experience simulations of situations in order to find and compare the experimental probability to the theoretical probability, students discover that rarely are those probabilities the same. Students often expect that simulations will result in all of the possibilities. All possibilities may occur in a simulation, but not necessarily. Theoretical probability does use all possibilities. Note examples in simulations when some possibilities are not shown Students often expect the theoretical and experimental probabilities of the same data to match. By providing multiple opportunities for students to experience simulations of situations in order to find and compare the experimental probability to the theoretical probability, students discover that rarely are those probabilities the same. Students often expect that simulations will result in all of the possibilities. All possibilities may occur in a simulation, but not necessarily. Theoretical probability does use all possibilities. Note examples in simulations when some possibilities are not shown		
Knowledge Students will know…	**Skills** Students will be able to…	

To continue their work with area from Grade 6, solving problems Involving the area and circumference of a circle and surface area of three-dimensional objects. To reason about relationships among two-dimensional figures using scale drawings and informal geometric constructions, and they gain familiarity with relationships between angles formed by Intersecting lines. To work with three-dimensional figures, relating them to two-dimensional figures by examining cross-sections. To solve real-world and mathematical problems involving area, surface area, and volume of two- and three-dimensional objects composed of triangles, quadrilaterals, polygons, cubes, and right prisms.	Draw, construct, and describe geometrical figures and describe the relationships between them Solve real-life and mathematical problems involving angle measure, area, surface area, and volume.

Assessment Evidence (Stage 2)

Performance Task Description

• **Goal** • **Role** • **Audience** • **Situation** • **Product/Performance** • **Standards**	**Topic A Calculating and Interpreting Probabilities** 7.SP.C.5 Understand that the probability of a chance event is a number between 0 and 1 that expresses the likelihood of the event occurring. Larger numbers indicate greater likelihood. A probability near 0 indicates an unlikely event, a probability around 1/2 indicates an event that is neither unlikely nor likely, and a probability near 1 indicates a likely event. 7.SP.C.6 Approximate the probability of a chance event by collecting data on the chance process that produces it and observing its long-run relative frequency, and predict the approximate relative frequency given the probability. For example, when rolling a number cube 600 times, predict that a 3 or 6 would be rolled roughly 200 times, but probably not exactly 200 times. 7.SP.C.7 Develop a probability model and use it to find probabilities of events. Compare probabilities from a model to observed frequencies; if the agreement is not good, explain possible sources of the discrepancy. a. Develop a uniform probability model by assigning equal probability to all outcomes, and use the model to determine probabilities of events. For example, if a student is selected at random from a class, find the probability that Jane will be selected and the probability that a girl will be selected. b. Develop a probability model (which may not be uniform) by observing frequencies in data generated from a chance process. For example, find the approximate probability that a spinning penny will land heads up or that a tossed paper cup will land open-end down. Do the outcomes for the spinning penny appear to be equally likely based on the observed frequencies? 7.SP.C.8 Find probabilities of compound events using organized lists, tables, tree diagrams, and simulation. a. Understand that, just as with simple events, the probability of a compound event is the fraction of outcomes in the sample space for which the compound event occurs. b. Represent sample spaces for compound events using methods such as organized lists, tables and tree diagrams. For an event described in everyday language (e.g., "rolling double sixes"), identify the outcomes in the sample space which compose the event Instructional Days: 7 Lesson 1: Chance Experiments (P)1 Lesson 2: Estimating Probabilities by Collecting Data (P) Lesson 3: Chance Experiments with Equally Likely Outcomes (P) Lesson 4: Calculating Probabilities for Chance Experiments with Equally Likely Outcomes (P) Lesson 5: Chance Experiments with Outcomes That Are Not Equally Likely (P) Lesson 6: Using Tree Diagrams to Represent a Sample Space and to Calculate Probabilities (P) Lesson 7: Calculating Probabilities of Compound Events (P) Topic B Estimating Probabilities Standard 7.SP.C.6 Approximate the probability of a chance event by collecting data on the chance process that produces it and observing its long-run relative frequency, and predict the approximate relative frequency given the probability. For example, when rolling a number cube 600 times, predict that a 3 or 6 would be rolled roughly 200

Other Evidence

Mid-Module Assessment Task After Topic B Constructed response with rubric 7.SP.C.5, 7.SP.C.6, 7.SP.C.7, 7.SP.C.8
End-of-Module Assessment Task After Topic D Constructed response with rubric 7.SP.A.1, 7.SP.A.2, 7.SP.B.3, 7.SP.B.4, 7.SP.C.5, 7.SP.C.6, 7.SP.C.7, 7.SP.C.8

Learning Plan (Stage 3)

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- **How will you organize and sequence the learning activities to optimize the engagement and achievement of ALL students?**

Students begin their study of probability, learning how to interpret probabilities and how to compute probabilities in simple settings. They also learn how to estimate probabilities empirically. The concept of probability provides a foundation for the thinking required to make inferential reasoning that is developed in the second half of this module. Additionally, students build on their knowledge of data distributions that they studied in Grade 6, compare data distributions of two or more populations, and are introduced to the idea of drawing informal inferences based on data collected from random samples. In Topics A and B, students learn to interpret the probability of an event as the proportion of the time that the event will occur when a chance experiment is repeated many times (7.SP.C.5). They learn to compute or estimate probabilities using a variety of methods, including collecting data, using tree diagrams, and using simulations. In Topic B, students move to comparing probabilities from simulations to computed probabilities that are based on theoretical models (7.SP.C.6, 7.SP.C.7). They calculate probabilities of compound events using lists, tables, tree diagrams, and simulations (7.SP.C.8). They learn to use probabilities to make decisions and to determine whether or not a given probability model is plausible (7.SP.C.7). The Mid-Module Assessment follows Topic B. In Topics C and D, students focus on using random sampling to draw informal inferences about a population (7.SP.A.1, 7.SP.A.2). In Topic C, they investigate sampling from a population (7.SP.A.2). They learn to estimate a population mean using numerical data from a random sample (7.SP.A.2). They also learn how to estimate a population proportion using categorical data from a random sample. In Topic D, students learn to compare two populations with similar variability. They learn to consider sampling variability when deciding if there is evidence that the means or the proportions of two populations are actually different (7.SP.B.3, 7.SP.B.4). The End-of-Module Assessment follows Topic D