

Title of Unit	The Number System	8th grade	Level 1
Curriculum Area	Mathematics	Time Frame	2-3 weeks
Developed By	Munira Jamali		
Identify Desired Results (Stage 1)			
Content Standards			
<p>7.NS.1 Apply and extend previous understandings of addition and subtraction to add and subtract rational numbers; represent addition and subtraction on a horizontal or vertical number line diagram. a. Describe situations in which opposite quantities combine to make 0. For example, a hydrogen atom has 0 charge because its two constituents are oppositely charged. b. Understand $p+q$ as the number located a distance from p, in the positive or negative direction depending on whether q is positive or negative. Show that a number and its opposite have a sum of 0 (are additive inverses). Interpret sums of rational numbers by describing real-world contexts. c. Understand subtraction of rational numbers as adding the additive inverse, $p-q=p+(-q)$. Show that the distance between two rational numbers on the number line is the absolute value of their difference, and apply this principle in real-world contexts. d. Apply properties of operations as strategies to add and subtract rational numbers.</p> <p>7.NS.2 Apply and extend previous understanding of multiplication and division and of fractions to multiply and divide rational numbers. a. Understand that multiplication is extended from fractions to rational numbers by requiring that operations continue to satisfy the properties of operations, particularly the distributive property, leading to products such as $(-1)(-1)=1$ and the rules for multiplying signed numbers. Interpret products of rational numbers by describing realworld contexts. b. Understand that integers can be divided, provided that the divisor is not zero, and every quotient of integers (with non-zero divisor) is a rational number. If p and q are integers, then $-(p/q)=(-p/q)=(p/-q)$. Interpret quotients of rational numbers by describing real-world contexts. c. Apply properties of operations as strategies to multiply and divide rational numbers. d. Convert a rational number to a decimal using long division; know that the decimal form of a rational number terminates in 0s or eventually repeats.</p> <p>7.NS.3 Solve real-world and mathematical problems involving the four operations with rational numbers.</p>			
Understandings		Essential Questions	
Overarching Understanding		Overarching	Topical

Computation with positive and negative numbers is often necessary to determine relationships between quantities. Models, diagrams, manipulatives, number lines, and patterns are useful in developing and remembering algorithms for computing with positive and negative numbers.

Properties of real numbers hold for all rational numbers. Positive and negative numbers are often used to solve problems in everyday life.

Demonstrate that a number and its opposite have a sum of 0. 6.

A positive quantity and negative quantity of the same absolute value add to make 0

When should we use additive inverse or multiplicative inverse?
 How do we use a number line to show addition and subtraction of rational numbers?
 What is the result of (what happens when) adding a number and its inverse or multiplying a number and its inverse?
 How is the identity related to its inverses?
 What is the relationship between addition and subtraction?
 What is the relationship between multiplication and

How do number properties assist in computation?
 Is estimation better way of understanding problem than finding an exact answer?
 How do we use ordinal numbers in everyday life?
 Where do we see numerals in the real world?

DIFFERENTIATION

FRONT LOADING

Skills of arithmetic for fractions, decimals and percents are required for introducing the concepts in this unit.

- Understanding of coordinate plane and graphing of linear functions will be useful in engaging students in the study of application of proportional relationships.
- Generate and solve linear equations • Understand solving formulas for different variables (t=pn; y=kx; i=prt)

ACCELERATION

How is rate of change related to the slope? • Multiple discounts • Limits of change • Rates of Change for Acceleration and Deceleration Use the following activities for acceleration: • First Rate (LEVEL D) - <http://insidemathematics.org/problems-of-themonth/pom-firstrate.pdf>

- 7.RP.2 Bagel Algebra - <http://illuminations.nctm.org/LessonDetail.aspx?id=L662>

INTERVENTION

Small teacher to student ratio discussion

- Emphasize think-pair-share
- Make connections to real life. Students understand that Part-to-part ratios are used to compare two parts. For example, the number of girls in the class (12) compared to the number of boys in the class (16) is the ratio the ratio 12 to 16.
- Illustrate the concept of ratios and proportions using real life examples. Continuing with the use of a table and graph, students can investigate and reason about proportions.
- Using kinesthetic activities and manipulatives

division?

How are the operations applied in real world contexts? How do the properties of operation help us compute with rational numbers?

Is it always true that multiplying a negative factor by a positive factor always produces a negative product

What do numerals represent?

What do numerals represent?

How do I demonstrate the relationship between numbers, quantities and place value for whole numbers up to 1,000

Knowledge

Students will know...

Skills

Students will be able to...

To extend addition, subtraction, multiplication, and division to all rational numbers, maintaining the properties of operations and the relationships between addition and subtraction, and multiplication and division. By applying these properties, and by viewing negative numbers in terms of everyday contexts (e.g., amounts owed or temperatures below zero),
To explain and interpret the rules for adding, subtracting, multiplying, and dividing with negative numbers. They use the arithmetic of rational numbers as they formulate expressions and equations in one variable and use these equations to solve problems.

Add and Subtract rational numbers
Multiply and divide fractions and extend it to rational numbers

Assessment Evidence (Stage 2)

Performance Task Description

- **Goal**
- **Role**
- **Audience**
- **Situation**
- **Product/Performance**
- **Standards**

Classifying Proportion and Non-Proportion Situations

<http://map.mathshell.org/lessons.php?collection=8&unit=7215#task483>

Students are able to:

- Identify when two quantities vary in direct proportion to each other.
- Distinguish between direct proportion and other functional relationships.
- Solve proportionality problems using efficient methods

7.RP: Analyze proportional relationships and use them to solve real-world and mathematical problems.

Increasing and Decreasing Quantities by a Percent

<http://map.mathshell.org/download.php?fileid=1623>

Students are able to

Translating between percents , decimals, and fractions.

- Representing percent increase and decrease as multiplication.
- Recognizing the relationship between increases and decreases

7.RP: Analyze proportional relationships and use them to solve real-world and mathematical problems.

7.NS: Apply and extend previous understandings of operations with fractions.

7.EE: Use properties of operations to generate equivalent expressions.

7. NS.1 and 7.NS.3 Using Positive and Negative Numbers in Context

<http://map.mathshell.org/materials/lessons.php?taskid=453#task453>

This lesson unit is intended to help you assess how well students are able to understand and use directed numbers in context. It is intended to help identify and aid students who have difficulties in ordering, comparing, adding, and subtracting positive and negative integers. Particular attention is paid to the use of negative numbers on number lines to explore the structures:

Starting temperature + Change in temperature = Final temperature

Final temperature – Change in temperature = Starting temperature

Final temperature – Starting temperature = Change in temperature

7.NS: Apply and extend previous understandings of operations with fractions. 7.EE: Use properties of operations to generate equivalent expressions. Solve real-life and mathematical problems using numerical and algebraic expressions and equations

Other Evidence

Illustrative Mathematics •
7.RP.1 Cooking with Whole Cup
<http://illustrativemathematics.org/illustrations/470>
7.RP.1 Track Practice
<http://illustrativemathematics.org/illustrations/8>
7RP.2 Art Class, Variations 1& Variations 2 - Buying Coffee
<http://illustrativemathematics.org/illustrations/100>
7.RP.2d Robot Races
<http://illustrativemathematics.org/illustrations/181>
7.RP.2 Sore Throats – Variation 1
<http://illustrativemathematics.org/illustrations/180>

Learning Plan (Stage 3)

- **Where** are your students headed? **Where** have they been? **How** will you make sure the students know where they are going?
- **How** will you **hook** students at the beginning of the unit?
- **What** events will help students **experience and explore** the big idea and questions in the unit? **How** will you equip them with needed skills and knowledge?
- **How** will you cause students to **reflect and rethink**? **How** will you guide them in rehearsing, revising, and refining their work?
- **How** will you help students to **exhibit and self-evaluate** their growing skills, knowledge, and understanding throughout the unit?
- **How** will you **tailor** and otherwise personalize the learning plan to optimize the engagement and effectiveness of ALL students, without compromising the goals of the unit?
- **How** will you **organize** and sequence the learning activities to optimize the engagement and achievement of ALL students?

Math Activities

Help students to gain a general understanding regarding adding integers on a number line that the sum $p+q$ is the number found when moving a total of $|q|$ units from p to the right if q is positive, and to the left if q is negative.

Use Number line model for operation with integers
Use of chips model (positive/negative numbers) for creating 0-pairs.

Use a foldable for integer rules.

Show that $|a+b| \neq |a| + |b|$. For example show that $(+12) + (-7)$ on a number line.

Students will describe situations in which opposite quantities will combine to make 0 or 1. Example:

To add -5 and 5, I _____.

The resulting sum will be _____, because _____.

• Students will explain how they will use the properties of operations to compute with rational numbers. Example: In performing operations with rational numbers, I will _____.

• Students will create/write real-world problems representing operations with rational numbers.

Example: If the temperature is 40 °F in the morning and increases by 100 °F by noon, the new temperature will be _____ because _____.

Title of Unit	Understand Expressions and Equations	Grade Level	8 th grade Level 1
Curriculum Area	Mathematics	Time Frame	3-4 weeks
Developed By	Munira Jamali		

Identify Desired Results (Stage 1)

Content Standards

7.EE.1.
Apply properties of operations as strategies to add, subtract, factor, and expand linear expressions with rational coefficients

7.EE.2.
Understand that rewriting an expression in different forms in a problem context can shed light on the problem and how the quantities in it are related. For example, $a + 0.05a = 1.05a$ means that "increase by 5%" is the same as "multiply by 1.05."

7.EE.3.
Solve multi-step real-life and mathematical problems posed with positive and negative rational numbers in any form (whole numbers, fractions, and decimals), using tools strategically. Apply properties of operations to calculate with numbers in any form; convert between forms as appropriate; and assess the reasonableness of answers using mental computation and estimation strategies. For example: If a woman making \$25 an hour gets a 10% raise, she will make an additional 1/10 of her salary an hour, or \$2.50, for a new salary of \$27.50. If you want to place a towel bar 9 3/4 inches long in the center of a door that is 27 1/2 inches wide, you will need to place the bar about 9 inches from each edge; this estimate can be used as a check on the exact computation.

7.EE.4.
Use variables to represent quantities in a real-world or mathematical problem, and construct simple equations and inequalities to solve problems by reasoning about the quantities.

a. Solve word problems leading to equations of the form $px + q = r$ and $p(x + q) = r$, where p , q , and r are specific rational numbers. Solve equations of these forms fluently. Compare an algebraic solution to an arithmetic solution, identifying the sequence of the operations used in each approach. For example, the perimeter of a rectangle is 54 cm. Its length is 6 cm. What is its width?

b. Solve word problems leading to inequalities of the form $px + q > r$ or $px + q < r$, where p , q , and r are specific rational numbers. Graph the solution set of the inequality and interpret it in the context of the problem. For example: As a salesperson, you are paid \$50 per week plus \$3 per sale. This week you want your pay to be at least \$100. Write

Understandings	Essential Questions	
Overarching Understanding	Overarching	Topical

<ul style="list-style-type: none"> • Generating equivalent, linear expressions with rational coefficients using the properties of operations will lead to solving linear equation. • Discovering that rewriting expressions in different forms in a problem context leads to understanding that the values are equivalent. • Ability to solve and explain real life and mathematical problems involving rational numbers using numerical and algebraic expressions is important for preparation for HS Algebra. • Constructing simple equations and inequalities to solve real life word problems is a necessary concept. • Write and solve real- life and mathematical problems involving simple equations for an unknown angle in a figure would help students as the engage in higher Geometry concepts. 	<p>How can I apply the order of operations and the fundamentals of algebra to solve problems? How can I justify that multiple representations in the context of a problem are equivalent expressions? How do I assess the reasonableness of my answer? How will I use the properties of equality to explain the order of the steps in solving equations and inequalities? How do I interpret the solutions for equations and inequalities in the context of the problem? How can I use and relate facts about special pairs of angles to write and solve simple equations involving unknown angles?</p>	<p>Do mathematical models conceal as much as they reveal? What patterns or relationships do we see in each type of mathematics? What are the different ways to represent the patterns or relationships? What different interpretations can be obtained from a particular pattern or relationship? What predictions can the patterns or relationships support? How can we use or test our predictions? Are they valid? Are they significant? Where in the real world would I find patterns?</p>
Differentiation		
<p>UDL/ FRONT LOADING Reason about and solve 1-variable equations and inequalities</p> <ul style="list-style-type: none"> • Apply and extend previous understandings of arithmetic to algebraic expressions • Apply and extend understandings of numbers to the number system of rational numbers <p>ACCELERATION Use the Building bridges activity to enrich high achieving students: http://illuminations.nctm.org/Lesson.aspx?id=952</p> <p>Intervention For low achieving students and students with disabilities:</p> <ul style="list-style-type: none"> • Small teacher to student ratio discussion • Emphasize think-pair-share • Make connections to real life • ALEKS –www.aleks.com • Small group re-teach • Using kinesthetic activities and manipulatives 		

Knowledge Students will know...	Skills Students will be able to...
<p>To develop a unified understanding of number, recognizing fractions, decimals (that have a finite or a repeating decimal representation), and percents as different representations of rational numbers.</p> <p>To extend addition, subtraction, multiplication, and division to all rational numbers division including expanding linear expressions with rational coefficient, maintaining the properties of operations and the relationships between addition and subtraction, and multiplication.</p> <p>To apply above properties, and by viewing negative numbers in terms of everyday contexts (e.g., amounts owed or temperatures below zero), students explain and interpret the rules for adding, subtracting, multiplying, and dividing with negative numbers. They use the arithmetic of rational numbers as they formulate expressions and equations in one variable and use these equations to solve problems</p>	<ul style="list-style-type: none"> • Apply properties of operations to generate equivalent expressions • Evaluate and solve real-life and mathematical problems using numerical and algebraic expressions and equations • Evaluate and solve real-life and mathematical problems involving angle measure, area, surface area, and volume
Assessment Evidence (Stage 2)	
Performance Task Description	

<ul style="list-style-type: none"> • Goal • Role • Audience • Situation • Product/Performance • Standards 	<p><u>Solving Linear Equations</u> http://map.mathshell.org/download.php?fileid=1635 This lesson unit is intended to help you assess how well students are able to:</p> <ul style="list-style-type: none"> • Form and solve linear equations involving factorizing and using the distributive law. <p>In particular, this unit aims to help you identify and assist students who have difficulties in:</p> <ul style="list-style-type: none"> • Using variables to represent quantities in a real-world or mathematical problem. <p>Solving word problems leading to equations of the form $px + q = r$ and $p(x + q) = r$.</p> <p>7.EE: Use properties of operations to generate equivalent expressions. Solve real-life and mathematical problems using numerical and algebraic expressions and equations.</p> <p><u>The wheel shop</u> www.insidemathematics.org/assets/problems-of-the-month/the%20wheel%20shop.pdf The goal is for all students to have the experience of attacking and solving non-routine problems and developing their mathematical reasoning skills. Although obtaining and justifying solutions to the problems is the objective, the process of learning to problem-solve is even more important</p> <p>7.EE.2. Understand that rewriting an expression in different forms in a problem context can shed light on the problem and how the quantities in it are related. For example, $a + 0.05a = 1.05a$ means that "increase by 5%" is the same as "multiply by 1.05"</p> <p>7.EE.4. Use variables to represent quantities in a real-world or mathematical problem, and construct simple equations and inequalities to solve problems by reasoning about the quantities.</p>
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Other Evidence

Illustrative Mathematics

- 7.EE.1– The Mango Problem

<http://illuminations.nctm.org/Lesson.aspx?id=1037>

- 7.EE.1– The Sailor and Coconut Problem

<http://illuminations.nctm.org/404.aspx?aspxerrorpath=/lessons/6-8/mangoes/Classic-AS-Sailor.pdf>

- 7.EE.1 and 7.EE.2 Pan Balance - Expressions

<http://illuminations.nctm.org/Lesson.aspx?id=2747>

- 7.EE.1 – Miles to Kilometers

<http://illustrativemathematics.org/illustrations/433>

- 7 EE.3 – Discounted Books

<http://illustrativemathematics.org/illustrations/478>

- 7.EE.4 and 4b. – Fishing Adventures 2

<https://www.illustrativemathematics.org/content-standards/tasks/643>

- 7 EE.4b – Sport Equipment Se

<http://www.illustrativemathematics.org/illustrations/986>

Learning Plan (Stage 3)

- **Where** are your students headed? **Where** have they been? **How** will you make sure the students know where they are going?
- **How** will you **hook** students at the beginning of the unit?
- **What** events will help students **experience** and **explore** the big idea and questions in the unit? **How** will you equip them with needed skills and knowledge?
- **How** will you cause students to **reflect** and **rethink**? **How** will you guide them in rehearsing, revising, and refining their work?
- **How** will you help students to **exhibit** and **self-evaluate** their growing skills, knowledge, and understanding throughout the unit?
- **How** will you **tailor** and otherwise personalize the learning plan to optimize the engagement and effectiveness of ALL students, without compromising the goals of the unit?
- **How** will you **organize** and sequence the learning activities to optimize the engagement and achievement of ALL students?

Math Activities

Help students to gain a fundamental understanding that the distributive property works "on the right" as well as "on the left," in addition to "forwards" as well as "backwards."

- Real-world connections (Use equations to set up a home budget, e.g. Percent of take-home pay for rent, utilities, food, savings, etc.to provide students a conceptual understanding of expressions and equations).
- Engage students in a discussion to learn that different ways of writing expressions can serve different purposes and provide different ways of seeing a problem. Have students use this example to work with expressions: A rectangle is twice as long as it is wide. Find as many different ways as you can to write an expression for the perimeter of such a rectangle.
- Use the Surround the Pool Concept Lesson <https://vcoms.wildapricot.org/resources/Documents/DOWNLOAD%20SURROUND%20THE%20POOL%20LP%20HER E.pdf>
- Structured instructional conversations (Think-Pair-Share)
- Peer Tutoring • Journal writing prompts - • Questioning Strategies

Students will evaluate the argument and specific claims in a word problem, including the validity of the reasoning, making explicit reference to words in the problem and using reporting language (According to the problem, ...; the problem states that...; the main points are...; argues, In my opinion, the way to solve this problem is...; What is most important in this problem is _____; because_____.

Students will read equations, expressions, and inequalities aloud fluently, without hesitating Students will summarize the steps in setting up and evaluating/solving expressions, equations and inequalities as described in their textbooks using the words first, second, third, etc.

Students will identify words, or phrases, in word problems that help them solve them using a causative structure such as: The following words "evaluate" and "solution," and "solution set" help me solve the problem

Students will use the definitions in their textbook to describe key geometrical shapes using the relative pronoun "whose" (angles whose measures add up to 180° are supplementary) Writing Students will write definitions of key vocabulary using complete, well-formed sentences.

Students will write a constructed response to a word problem using logically ordered reasons that are supported by facts and details and using the appropriate mathematic vocabulary

Students will list possible reasons for their conclusions, using

Title of Unit	Ratios and Proportion	Grade Level	8 th grade Level1
Curriculum Area	Mathematics	Time Frame	3-4 weeks
Developed By	Munira Jamali		
Identify Desired Results (Stage 1)			
Content Standards			

7.RP.1 Ratios and Proportion

Compute unit rates associated with ratios of fractions, including ratios of lengths, areas and other quantities measured in like or different units.

For example, if a person walks $\frac{1}{2}$ mile in each $\frac{1}{4}$ hour, compute the unit rate as the complex fraction $\frac{1/2}{1/4}$ miles per hour, equivalently 2 miles per hour.

7.RP.2 Recognize and represent proportional relationships between quantities.

a. Decide whether two quantities are in a proportional relationship, e.g., by testing for equivalent ratios in a table or graphing on a coordinate plane and observing whether the graph is a straight line through the origin.

b. Identify the constant of proportionality (unit rate) in tables, graphs, equations, diagrams, and verbal descriptions of proportional relationships.

c. Represent proportional relationships by equations.

For example, if total cost t is proportional to the number n of items purchased at a constant price p , the relationship between the total cost and the number of items can be expressed as $t = pn$. d. Explain what a point (x, y) on the graph of a proportional relationship means in terms of the situation, with special attention to the points $(0, 0)$ and $(1, r)$ where r is the unit rate.

7.RP.3

Use proportional relationships to solve multistep ratio and percent problems.

Examples: simple interest, tax, markups and markdowns, gratuities and commissions, fees, percent increase and decrease, percent error.

7.G.1 Geometry

Solve problems involving scale drawings of geometric figures, including computing actual lengths and areas from a scale drawing and reproducing a scale drawing at a different scale.

7.G.2

Draw (freehand, with ruler and protractor, and with technology) geometric shapes with given conditions. Focus on constructing triangles from three measures of angles or sides, noticing when the conditions determine a unique triangle, more than one triangle, or no triangle.

Understandings	Essential Questions	
Overarching Understanding	Overarching	Topical

Proportional reasoning is essential in problem solving

- Understanding mathematical relationships allows us to make predictions, calculate and model unknown quantities.
- Proportional relationships express how quantities change in relationship to each other. (Look at NYC Common Core Grade 7)

Reason about relationships among two-dimensional figures using scale drawings and informal geometric constructions, which will lead to gaining familiarity with the relationships between angles formed by intersecting lines. Work with three-dimensional figures, relating them to two-dimensional figures by examining cross-sections.

- How can proportions be used to solve problems?
- When is a relationship proportional?
- How can proportions increase our understanding of the real world?
- How does the mathematical use of the word similar differ from the everyday use?
- How can similarity help us solve measurement problems?

Why are ratios and proportions important??
 Does my answer make sense?
 How can I compare two different things? (How can we show the relationship between two quantities or values?)
 How should I solve it? (What number form should be used to solve a

DIFFERENTIATION

FRONT LOADING ACCELERATION INTERVENTION

- Skills of arithmetic for fractions, decimals and percents are required for introducing the concepts in this unit.
- Understanding of coordinate plane and graphing of linear functions will be useful in engaging students in the study of application of proportional relationships.
- Generate and solve linear equations
- Understand solving formulas for different variables ($t=pn$; $y=kx$; $i=prt$)

ACCELERATION

- Skills of arithmetic for fractions, decimals and percents are required for introducing the concepts in this unit.
- Understanding of coordinate plane and graphing of linear functions will be useful in engaging students in the study of application of proportional relationships.
- Generate and solve linear equations
- Understand solving formulas for different variables ($t=pn$; $y=kx$; $i=prt$)
- How is rate of change related to the slope?
- Multiple discounts • Limits of change
- Rates of Change for Acceleration and Deceleration Use the following activities for acceleration:
 - First Rate (LEVEL D) - <http://insidemathematics.org/problems-of-themonth/pom-firstrate.pdf>
 - 7.RP.2 Bagel Algebra - <http://illuminations.nctm.org/LessonDetail.aspx?id=L662>

INTERVENTION

Small teacher to student ratio discussion

- Emphasize think-pair-share
- Make connections to real life. Students understand that Part-to-part ratios are used to compare two parts. For example, the number of girls in the class (12) compared to the number of boys in the class (16) is the ratio the ratio 12 to 16.
- Illustrate the concept of ratios and proportions using real life examples. Continuing with the use of a table and graph, students can investigate and reason about proportions.
- Using kinesthetic activities and manipulatives

- What are the connections between similarity, geometry and algebra? What is the total number of degrees in supplementary and complementary angles? What is the relationship between vertical and adjacent Angles?

problem?)

Knowledge

Students will know...

Skills

Students will be able to...

To extend their understanding of ratios and develop understanding of proportionality to solve single- and multi-step problems.

To use their understanding of ratios and proportionality to solve a wide variety of percent problems, including those involving discounts, interest, taxes, tips, and percent increase or decrease.

To solve problems about scale drawings by relating corresponding lengths between the objects or by using the fact that relationships of lengths within an object are preserved in similar objects.

To graph proportional relationships and understand the unit rate informally as a measure of the steepness of the related line, called the slope.

To distinguish proportional relationships from other relationships

To reason about relationships among two-dimensional figures using scale drawings and informal geometric constructions, and they gain familiarity with relationships between angles formed by intersecting lines.

Analyze proportional relationships and use them to solve real-world and mathematical problem

Draw, construct, and describe geometrical figures and describe the relationships between them

Make sense of problems and persevere in solving them.

Reason abstractly and quantitatively

To Draw, construct, and describe geometrical figures and describe the relationships between them.

Assessment Evidence (Stage 2)

Performance Task Description

- **Goal**
- **Role**
- **Audience**
- **Situation**
- **Product/Performance**
- **Standards**

7.RP.1 and 7.RP.2 Proportion and Non-proportion Situations
<http://map.mathshell.org/materials/lessons.php?taskid=483#task483>

This lesson unit is intended to help you assess whether students are able to:

- Identify when two quantities vary in direct proportion to each other.
- Distinguish between direct proportion and other functional relationships.
- Solve proportionality problems using efficient methods

Standard

7.RP: Analyze proportional relationships and use them to solve real-world and mathematical problems

7.RP.1 and 7.G.1 Developing a Sense of Scale

<http://map.mathshell.org/download.php?fileid=1631>

This lesson unit is intended to help you assess whether students recognize relationships of direct proportion and how well they solve problems that involve proportional reasoning. In particular, it is intended to help you identify those students who:

- Use inappropriate additive strategies in scaling problems, which have a multiplicative structure.
- Rely on piecemeal and inefficient strategies such as doubling, halving, and decomposition and have not developed a single multiplier strategy for solving proportionality problems.
- See multiplication as making numbers bigger and division as making numbers smaller

7.RP: Analyze proportional relationships and use them to solve real-world and mathematical problems

7.RP.3 Increasing and Decreasing Quantities by a Percent

<http://map.mathshell.org/lessons.php?unit=7100&collection=8&redir=1>

This lesson unit is intended to help you assess how well students are able to interpret percent increase and decrease and in particular, to identify and help students who have the following difficulties:

- Translating between percents, decimals, and fractions.
- Representing percent increase and decrease as multiplication.
- Recognizing the relationship between increases and decrease

7.RP: Analyze proportional relationships and use them to solve real-world and mathematical problems.

7.NS: Apply and extend previous understandings of operations with fractions.

7.EE: Use properties of operations to generate equivalent expressions

Drawing to Scale: A Garden

<http://map.mathshell.org/download.php?fileid=1641>

This lesson unit is intended to help assess how well students are able to interpret and use scale drawings to plan a garden layout. This involves using proportional reasoning and metric units.

Standards

Mathematical Content in the Common Core State Standards for Mathematics:

7.G: Draw, construct, and describe geometrical figures and describe the relationships between them. Solve real-life and mathematical

Other Evidence

Illustrative Mathematics

7.RP.1 Molly's Run

www.illustrativemathematics.org/content-standards/tasks/828

7.RP.2 Music Companies, Variations 1

<https://www.illustrativemathematics.org/content-standards/tasks/95>

7.RP.1 Cooking with Whole Cup

<https://www.illustrativemathematics.org/content-standards/tasks/470>

- 7.RP.1 Track Practice

<https://www.illustrativemathematics.org/content-standards/tasks/82>

- 7 RP.2 Art Class, Variations 1& Variations 2 - Buying Coffee

<https://www.illustrativemathematics.org/content-standards/tasks/104>

7.RP.2d Robot Races 7.RP.2 Sore Throats – Variation

<https://www.illustrativemathematics.org/content-standards/tasks/181>

Learning Plan (Stage 3)

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- **How** will you **hook** students at the beginning of the unit?
- **What** events will help students **experience and explore** the big idea and questions in the unit? **How** will you equip them with needed skills and knowledge?
- **How** will you cause students to **reflect and rethink**? **How** will you guide them in rehearsing, revising, and refining their work?
- **How** will you help students to **exhibit and self-evaluate** their growing skills, knowledge, and understanding throughout the unit?
- **How** will you **tailor** and otherwise personalize the learning plan to optimize the engagement and effectiveness of ALL students, without compromising the goals of the unit?
- **How** will you **organize** and sequence the learning activities to optimize the engagement and achievement of ALL students?

Math Activities

Real-world connections (e.g. Use grocery store ads to find unit rates for various products)

- Structured instructional conversations (Think-Pair-Share)
- Peer Tutoring
- Journal writing prompts (link)
- Use visuals to illustrate multiple representations of rate of change.

Students will evaluate the argument and specific claims in a word problem, including the validity of the reasoning, making explicit reference to words in the problem and using reporting language (According to the problem, ...; the problem states that...; the main points are...; argues, In my opinion, the way to solve this problem is...; What is most important in this problem is _____; because_____.

Students will read ratios, proportions, and percent's aloud fluently, without hesitating. Students will summarize the steps in setting up and solving a proportion as described in their textbooks using the words first, second, third, etc. Students will identify words, or phrases, in word problems that help them solve them using a causative structure such as: The following words "unit " and "rate" help me solve the problem

Writing

Students will write definitions of key vocabulary using complete, well-formed sentences.

Students will write a constructed response to a word problem using logically ordered reasons that are supported by facts and details and using the appropriate mathematic vocabulary.

Students will list possible reasons for their conclusions, using verbs such as explain, demonstrate, justify and because).

Students will explain how they use a specific mathematical concept in their lives, using the following specific set of words: miles per gallon, miles per hour, feet per second, cents/pound, "the ratio of a to b." Listening and Speaking Students will explain how to set up and solve a proportion to a partner using the words first, second, third, etc.

Students will describe the relationship between fraction, ratio, proportion, using the words comparison, part to whole, part to part

Title of Unit	Geometry	Grade Level	8 th grade Level 1
Curriculum Area	Mathematics	Time Frame	2-3 weeks
Developed By	Munira Jamali		
Identify Desired Results (Stage 1)			
Content Standards			

Equations and Expressions

7.EE.3

Solve multi-step real-life and mathematical problems posed with positive and negative rational numbers in any form (whole numbers, fractions, and decimals), using tools strategically. Apply properties of operations to calculate with numbers in any form; convert between forms as appropriate; and assess the reasonableness of answers using mental computation and estimation strategies. For example: If a woman making \$25 an hour gets a 10% raise, she will make an additional $\frac{1}{10}$ of her salary an hour, or \$2.50, for a new salary of \$27.50. If you want to place a towel bar $9\frac{3}{4}$ inches long in the center of a door that is $27\frac{1}{2}$ inches wide, you will need to place the bar about 9 inches from each.

7.EE.4

Use variables to represent quantities in a real-world or mathematical problem, and construct simple equations and inequalities to solve problems by reasoning about the quantities.

c. Solve word problems leading to equations of the form $px + q = r$ and $p(x + q) = r$, where p , q , and r are specific rational numbers. Solve equations of these forms fluently. Compare an algebraic solution to an arithmetic solution, identifying the sequence of the operations used in each approach. For example, the perimeter of a rectangle is 54 cm. Its length is 6 cm. What is its width?

d. Solve word problems leading to inequalities of the form $px + q > r$ or $px + q < r$, where p , q , and r are specific rational numbers. Graph the solution set of the inequality and interpret it in the context of the problem. For example: As a salesperson, you are paid \$50 per week plus \$3 per sale. This week you want your pay to be at least \$100. Write an inequality for the number of sales you need to make, and describe the solutions.

Geometry

7.G.3

Describe the two-dimensional figures that result from slicing three dimensional figures, as in plane sections of right rectangular prisms and right rectangular pyramids.

7.G.5

Use facts about supplementary, complementary, vertical, and adjacent angles in a multi-step problem to write and solve simple equations for an unknown angle in a figure. Solve real-life and mathematical problems involving angle measure, area, surface area, and volume.

7.G.4

Know the formulas for the area and circumference of a circle and use them to solve problems; give an informal **derivation of the relationship between the circumference and area of a circle.**

7.G.6

Solve real-world and mathematical problems involving area, volume and surface area of two- and three-dimensional objects composed of triangles, quadrilaterals, polygons, cubes, and right prisms

Understandings		Essential Questions	
Overarching Understanding		Overarching	Topical

Constructing simple equations and inequalities to solve real life word problems is a necessary concept.

- Write and solve real- life and mathematical problems involving simple equations for an unknown angle in a figure would help students as the engage in higher Geometry concepts. Solve problems involving the area and circumference of a circle and surface area of three dimensional objects.
- Reason about relationships among two-dimensional figures using scale drawings and informal geometric constructions, which will lead to gaining familiarity with the relationships between angles formed by intersecting lines. Work with three-dimensional figures, relating them to two- dimensional figures by examining cross-sections.
- Solve real-world and mathematical problems involving area, surface area, and volume of two- and three-dimensional objects composed of triangles, quadrilaterals, polygons, cubes and right prisms.

How do I assess the reasonableness of my answer?

How will I use the properties of equality to explain the order of the steps in solving equations and inequalities?

How do I interpret the solutions for equations and inequalities in the context of the problem?

What 2-D figure results from slicing 3-D figures?

(cones, spheres, or cylinders) How do you find the surface area and volume of a 3D figure?

What is the total number of degrees in supplementary and complementary angles?

What is the relationship between vertical and adjacent angles?

How would the volume and surface area be affected when dimensions of a figure are doubled and/or triple?

Where in the real world can I find shapes?

Where would you find symmetry?

How can objects be represented and compared using geometric attributes?

Is geometry more like map-making and using a map, or inventing and playing games like chess?

How can I identify and describe solid figures by describing the faces, edges, and sides?

DIFFERENTIATION

UDL/ FRONT LOADING

Students work on problems involving areas and volumes. • Students understand multiple algorithms for the volume of prisms • Students apply visualization skills connected to solve the area of 3D shapes. • Students can construct 3d models from 2d models.

Acceleration for high achieving students

Extension of standard 7G.3 - Students describe or define the features or characteristics of 2-D geometric figures that result when 3d figures are sliced horizontally, vertically or diagonally.

Intervention for low achieving students and students with disabilities:

- Small teacher to student ratio discussion – have students draw informal comparative inferences about two populations (boys vs. girls)
 - Data discussed comes from sampling life data (soccer team height vs. football team height)
 - Use Physical objects to demonstrate the math.
- In geometry : Such as cones, squares, sphere, etc.

Knowledge

Students will know...

To continue their work with area from Grade 6, solving problems Involving the area and circumference of a circle and surface area of three-dimensional objects.
 To reason about relationships among two-dimensional figures using scale drawings and informal geometric constructions, and they gain familiarity with relationships between angles formed by Intersecting lines.
 To work with three-dimensional figures, relating them to two-dimensional figures by examining cross-sections.
 To solve real-world and mathematical problems involving area, surface area, and volume of two- and three-dimensional objects composed of triangles, quadrilaterals, polygons, cubes, and right prisms.

Skills

Students will be able to...

Draw, construct, and describe geometrical figures and describe the relationships between them
 Solve real-life and mathematical problems involving angle measure, area, surface area, and volume.

Assessment Evidence (Stage 2)**Performance Task Description**

<ul style="list-style-type: none"> • Goal • Role • Audience • Situation • Product / Performance • Standards 	<p>7.G.6 Maximizing Area: Gold Rush http://map.mathshell.org/download.php?fileid=1637 Maximizing Area: Gold Rush This lesson unit is intended to help you assess how well students are able to:</p> <ul style="list-style-type: none"> • Interpret a situation and represent the variables mathematically. • Select appropriate mathematical methods to use and communicate their reasoning clearly. • Explore the effects on a rectangle's area of systematically varying the dimensions whilst keeping the perimeter constant. Interpret and evaluate the data generated, identifying the optimum case <p>Standards 7.G: Draw, construct, and describe geometrical figures and describe the relationships between them. Solve real-life and mathematical problems involving angle measure, area, surface area, and volume. 7.EE: Solve real-life and mathematical problems using numerical and algebraic expressions and equations.</p> <p>Using Dimensions: Designing a Sports Bag http://moodle.wbrschools.net/pluginfile.php/3902/mod_resource/content/1/using_dimensions_designing_a_sports_bag_complete.pdf This lesson unit is intended to help you assess how well students are able to:</p> <ul style="list-style-type: none"> • Recognize and use common 2D representations of 3D objects. • Identify and use the appropriate formula for finding the circumference of a circle. <p>Standards 7.G: Draw, construct and describe geometrical figures and describe the relationships between them. Solve real-life and mathematical problems involving angle measure, area, surface area, and volume. Drawing to Scale: A Garden http://map.mathshell.org/download.php?fileid=1641 This lesson unit is intended to help assess how well students are able to interpret and use scale drawings to plan a garden layout. This involves using proportional reasoning and metric units.</p> <p>Standards 7.G: Draw, construct, and describe geometrical figures and describe the relationships between them. Solve real-life and mathematical problems involving angle measure, area, surface area, and volume. 7.EE: Solve real-life and mathematical problems using numerical and algebraic expressions and equations. 7.RP: Analyze proportional relationships and use them to solve real-world and mathematical problems.</p> <p>7.G.6 Estimations and Approximations: The Money Munchers http://map.mathshell.org/materials/lessons.php?taskid=220#task220 This lesson unit is intended to help you assess how well students are able to:</p> <ul style="list-style-type: none"> • Model a situation. • Make sensible, realistic assumptions and estimates. • Use assumptions and estimates to create a chain of reasoning, in order to solve a practical problem. <p>7.G: Solve real-life and mathematical problems involving angle measure, area, surface area, and volume</p>
<p>Other Evidence</p>	

Illustrative Mathematics
7.SP Estimating the Mean State Area
<https://www.illustrativemathematics.org/illustrations/260>
Engage NY Common Core Curriculum
Module 6 – Geometry

Learning Plan (Stage 3)

- **Where** are your students headed? Where have they been? How will you make sure the students know where they are going?
- How will you **hook** students at the beginning of the unit?
- What events will help students **experience and explore** the big idea and questions in the unit? How will you equip them with needed skills and knowledge?
- How will you cause students to **reflect and rethink**? How will you guide them in rehearsing, revising, and refining their work?
- How will you help students to **exhibit and self-evaluate** their growing skills, knowledge, and understanding throughout the unit?
- How will you **tailor** and otherwise personalize the learning plan to optimize the engagement and effectiveness of ALL students, without compromising the goals of the unit?
- How will you **organize** and sequence the learning activities to optimize the engagement and achievement of ALL students?

Math Activities

Journal writing prompts (link)

- Technology to show visual representations of geometric figures: Geometry sketchpad
- Use visuals to illustrate multiple representations of rate of change • Real-world connections
- Structured instructional conversations (Think-Pair-Share)

Students will understand that some verbs have different meanings in different mathematical situations. (draw)

- Students will be able to interpret the characteristics of 2D and 3D figures in order to manipulate them. Example: The difference between 2D figure and 3D figure is _____.

Students will be able to select the appropriate formulas needed to solve real-world and mathematical problems. Example: I can compare the formulas for computing area, surface area, and volume of figures and objects, by _____.

Title of Unit	Statistics and Probability	Grade Level	8 th grade Level 1
Curriculum Area	Mathematics	Time Frame	3-4 weeks
Developed By	Munira Jamali		
Identify Desired Results (Stage 1)			
Content Standards			

7.SP.1.

Understand that statistics can be used to gain information about a population by examining a sample of the population; generalizations about a population from a sample are valid only if the sample is representative of that population. Understand that random sampling tends to produce representative samples and support valid inferences.

7.SP.2

Use data from a random sample to draw inferences about a population with an unknown characteristic of interest. Generate multiple samples (or simulated samples) of the same size to gauge the variation in estimates or predictions. For example, estimate the mean word length in a book by randomly sampling words from the book; predict the winner of a school election based on randomly sampled survey data. Gauge how far off the estimate or prediction might be.

7.SP.3.

Informally assess the degree of visual overlap of two numerical data distributions with similar variabilities, measuring the difference between the centers by expressing it as a multiple of a measure of variability. For example, the mean height of players on the basketball team is 10 cm greater than the mean height of players on the soccer team, about twice the variability (mean absolute deviation) on either team; on a dot plot, the separation between the two distributions of heights is noticeable. 7.SP.4. Use measures of center and measures of variability for numerical data from random samples to draw informal comparative inferences about two populations. For example, decide whether the words in a chapter of a seventh-grade science book are generally longer than the words in a chapter of a fourth-grade science book. (s/a)² Investigate chance processes and develop, use, and evaluate probability models.

7.SP.5.

Understand that the probability of a chance event is a number between 0 and 1 that expresses the likelihood of the event occurring. Larger numbers indicate greater likelihood. A probability near 0 indicates an unlikely event, a probability around 1/2 indicates an event that is neither unlikely nor likely, and a probability near 1 indicates a likely event.

7.SP.6.

Approximate the probability of a chance event by collecting data on the chance process that produces it and observing its long-run relative frequency, and predict the approximate relative frequency given the probability. For example, when rolling a number cube 600 times, predict that a 3 or 6 would be rolled roughly 200 times, but probably not exactly 200 times.

7.SP.7.

Develop a probability model and use it to find probabilities of events. Compare probabilities from a model to observed frequencies; if the agreement is not good, explain possible sources of the discrepancy. a. Develop a uniform probability model by assigning equal probability to all outcomes, and use the model to determine probabilities of events. For example, if a student is selected at random from a class, find the probability that Jane will be selected and the probability that a girl will be selected. b. Develop a probability model (which may not be uniform) by observing frequencies in data generated from a chance process. For example, find the approximate probability that a spinning penny will land heads up or that a tossed paper cup will land open-end down. Do the outcomes for the spinning penny appear to be equally likely based on the observed frequencies?

7.SP.8.

Find probabilities of compound events using organized lists, tables, tree diagrams, and

Understandings	Essential Questions	
Overarching Understanding	Overarching	Topical
<ul style="list-style-type: none"> • Compare two data distributions and address questions about differences between populations. • Begin informal work with random sampling to generate data sets and learn about the importance of representative samples for drawing inferences. 	<p>How do you determine which measures of variability should be used to draw informal comparative inferences? How are lists, tables, tree diagrams or simulation used to find the probability of an event? How is probability used to predict frequency of</p>	<p>When solving multi-step word problems using charts, tables, and graphs, how can you tell if the information is sufficient?</p>
DIFFERENTIATION		

UDL/ FRONT LOADING

Statistics and Probability:

- Students Formulate questions that can be answered with data
 - Students design and use a plan to collect relevant data
 - Students analyze the data with appropriate methods
 - Students are able to interpret results and draw valid conclusions from the data that relate to the questions posed. •
- Such investigations involve making sense of practical problems by turning them into statistical investigations; moving from context to abstraction and back to context; repeating the process of statistical reasoning in a variety of contexts.

Acceleration for high achieving students:

Provide students with opportunities to be recognized for their previous knowledge and to be allowed to avoid redundant learning by being encouraged to learn the sophisticated and advanced information and skills of the curriculum at their own rate. This also includes the opportunity for students to make personal meaning of the lesson.

Statistics and Probability:

Simulating probability experiments via technology where students collect meaningful data (type of music, who eats cafeteria food). Use the following activity for acceleration: Election Poll, Variation 3 <http://www.illustrativemathematics.org/illustrations/558>

Intervention for low achieving students and students with disabilities:

- Small teacher to student ratio discussion – have students draw informal comparative inferences about two populations (boys vs. girls)
- Data discussed comes from sampling life data (soccer team height vs. football team height)

an event?

When using a specific mean, median, you calculate the arithmetic mean?

How do you collect data?

How can you collect, organize, and display data?

How do you interpret the data you have collected?

What information does a chart or table give? How do charts, tables, and graphs help you interpret data?

Knowledge

Students will know...

Skills

Students will be able to...

To build on their work with single data distributions to compare two data distributions and address questions about differences between populations.

To begin informal work with random sampling to generate data sets and learn about the importance of representative samples for drawing inference

Use random sampling to draw inferences about a population. Draw informal comparative inferences about two populations.

Investigate chance processes and develop, use, and evaluate probability models.

Look for and express regularity in repeated reasoning.

Assessment Evidence (Stage 2)

Performance Task Description

- **Goal**
- **Role**
- **Audience**
- **Situation**
- **Product/Performance**
- **Standards**

7.SP.1 Estimating: Counting Tree

<http://map.mathshell.org/materials/lessons.php?taskid=422#task422>

This lesson unit is intended to help you assess how well students are able to:

- Solve simple problems involving ratio and direct proportion.
- Choose an appropriate sampling method.
- Collect discrete data and record them using a frequency table.

Standards

7.RP: Analyze proportional relationships and use them to solve real-world and mathematical problems.

7.G: Solve real-life and mathematical problems involving angle measure, area, surface area, and volume.

7.SP: Use random sampling to draw inferences about a population.

7.SP.5-8 Evaluating Statements About Probability

<http://map.mathshell.org/materials/lessons.php?taskid=220#task220>

This lesson unit addresses common misconceptions relating to the probability of simple and compound events. The lesson will help you assess how well students understand concepts of:

- Equally likely events.
- Randomness.
- Sample size.

Other Evidence

7.SP.2 and &.SP.7 Election Poll, Variation 1
<http://www.illustrativemathematics.org/illustrations/235>
 • 7.SP.2 and SP.2 Election Poll, Variation 2
<http://www.illustrativemathematics.org/illustrations/559/>

Learning Plan (Stage 3)

- **Where** are your students headed? **Where have they been? How will you make sure the students know where they are going?**
- **How will you hook** students at the beginning of the unit?
- **What events will help students experience and explore** the big idea and questions in the unit? **How will you equip them with needed skills and knowledge?**
- **How will you cause students to reflect and rethink?** How will you guide them in rehearsing, revising, and refining their work?
- **How will you help students to exhibit and self-evaluate** their growing skills, knowledge, and understanding throughout the unit?
- **How will you tailor and otherwise personalize the learning plan to optimize the engagement and effectiveness of ALL students, without compromising the goals of the unit?**
- **How will you organize and sequence the learning activities to optimize the engagement and achievement of ALL students?**

Journal writing prompts (link)

- Technology to show visual representations of geometric figures: Geometry sketchpad
 - Use visuals to illustrate multiple representations of rate of change
 - Real-world connections
 - Structured instructional conversations (Think-Pair-Share)
- LANGUAGE GOALS for low achieving, high achieving, students with disabilities and English Language Learners

Students will understand the context and relationship between data in order to make prediction and draw inferences. Example: Given two different sets of data, I can predict that _____. Based on this prediction, I could draw inference that _____. Students will be able to justify steps taken to arrive at a logical conclusion. Example: If the situation is _____, then I conclude that _____. P