Pre-Calculus

	Key Standards Covered	Possible Resources

*Optional

Quarter 1 September 6- November 2	• CCSS.MATH.CONTENT.HSA.CED.A.2: Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales. • CCSS.MATH.CONTENT.HSA.REI.D.10: Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line). • HSF.IF.A.1: Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If f is a function and x is an element of its domain, then f(x) denotes the output of f corresponding to the input x. The graph of f is the graph of the equation y = f(x). • HSF.IF.B.4: For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.* • HSF.IF.B.6: Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.* • HSF.IF.C.7a: Graph linear and quadratic functions	• AIT Project #1- Coordinate Plane String Art: Students will choose one of 15 designs, then given a cardstock square that has a 4-quadrant coordinate plane on the back; they will follow the sets of ordered pairs, sewing through each plotted point in order to create elaborate string art. • https://d3jc3ahdjad7x7.cloudfront.net/DlFncUvTNcS8RpquyagOjyoymt31dVVyFETqwfmISGkCv7we.pdf • AIT Project #2- Stained Glass Graphing: Students will practice graphing lines in slope-intercept form. They first must make sure the given equations are in the correct form then graph each linear equation. They will be graphing lines with positive, negative, zero, and undefined slopes. When completed, the correctly graphed lines will created a unique pattern that they will then use (markers/

*Optional

	Key Standards Covered	Possible Resources

*Optional

Quarter 2 November 12- January 28	• CCSS.MATH.CONTENT.HSA.CED.A.2: Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales. • CCSS.MATH.CONTENT.HSA.REI.D.10: Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line). • HSF.IF.A.1: Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If f is a function and x is an element of its domain, then f(x) denotes the output of f corresponding to the input x. The graph of f is the graph of the equation y = f(x). • HSF.IF.B.4: For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.* • HSF.IF.B.6: Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.* • HSF.IF.C.7a: Graph linear and quadratic functions	• AIT Project #1- Coordinate Plane String Art: Students will choose one of 15 designs, then given a cardstock square that has a 4-quadrant coordinate plane on the back; they will follow the sets of ordered pairs, sewing through each plotted point in order to create elaborate string art. • https://d3jc3ahdjad7x7.cloudfront.net/DlFncUvTNcS8RpquyagOjyoymt31dVVyFETqwfmISGkCv7we.pdf • AIT Project #2- Stained Glass Graphing: Students will practice graphing lines in slope-intercept form. They first must make sure the given equations are in the correct form then graph each linear equation. They will be graphing lines with positive, negative, zero, and undefined slopes. When completed, the correctly graphed lines will created a unique pattern that they will then use (markers/

*Optional

	Key Standards Covered	Possible Resources

*Optional

Quarter 3
February 4- April 5
	• CCSS.MATH.CONTENT.HSN.RN.A.1
	  • Explain how the definition of the meaning of rational exponents follows from extending the properties of integer exponents to those values, allowing for a notation for radicals in terms of rational exponents.
	• CCSS.MATH.CONTENT.HSN.RN.A.2
	Rewrite expressions involving radicals and rational exponents using the properties of exponents
	• CCSS.MATH.CONTENT.HSN.RN.B.3
	  • Explain why the sum or product of two rational numbers is rational; that the sum of a rational number and an irrational number.
	• CCSS.MATH.CONTENT.HSA.APR.A.1
	  • Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials.
	• CCSS.MATH.CONTENT.HSA.APR.B.2
	  • Know and apply the Remainder Theorem: For a polynomial $p(x)$ and a number a, the remainder on division by $x - a$ is $p(a)$, so $p(a) = 0$ if and only if $(x - a)$ is a factor of $p(x)$.
		• AIT Project #1 - Laws of Exponents Foldable Study Guide : Students will construct a foldable flipbook to use a study guide and learning manipulative for the rules and laws of exponents.
		• AIT Project #2 - The Size of our Solar System: Students will create a scaled model of our solar system out of their choice of materials. They must include the planets measurements and distances all using scientific notation. They will be given conversions and other reference materials from the NasaMath workbook to complete this task.
		• Working with scientific notation, students will solve problems about mass and percentages.

*Optional

	Key Standards Covered	Possible Resources

*Optional

Quarter 4
April 8- June 17
	• CCSS.MATH.CONTENT.HSN.RN.A.1
	  • Explain how the definition of the meaning of rational exponents follows from extending the properties of integer exponents to those values, allowing for a notation for radicals in terms of rational exponents.
	• CCSS.MATH.CONTENT.HSN.RN.A.2
	Rewrite expressions involving radicals and rational exponents using the properties of exponents
	• CCSS.MATH.CONTENT.HSN.RN.B.3
	  • Explain why the sum or product of two rational numbers is rational; that the sum of a rational number and an irrational number.
	• CCSS.MATH.CONTENT.HSA.APR.A.1
	  • Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials.
	• CCSS.MATH.CONTENT.HSA.APR.B.2
	  • Know and apply the Remainder Theorem: For a polynomial $p(x)$ and a number a, the remainder on division by $x - a$ is $p(a)$, so $p(a) = 0$ if and only if $(x - a)$ is a factor of $p(x)$.
		• AIT Project #1 - Laws of Exponents Foldable Study Guide : Students will construct a foldable flipbook to use a study guide and learning manipulative for the rules and laws of exponents.
		• AIT Project #2 - The Size of our Solar System: Students will create a scaled model of our solar system out of their choice of materials. They must include the planets measurements and distances all using scientific notation. They will be given conversions and other reference materials from the NasaMath workbook to complete this task.
		• Working with scientific notation, students will solve problems about mass and percentages.

*Optional

Title of Unit	Trigonometric Functions	Grade Level	10, 11, 12
Curriculum Area	PreCalculus	Time Frame	3-4 Weeks
Developed By	Douglas Frost		
Identify Desired Results (Stage 1)			
Content Standards			

*Optional

A-CED.2 Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.
A-REI.10 Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line).
F-IF.7.b Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions.
F-IF.7.e Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude.
F-IF.8.a Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context.
F-BF.3 Identify the effect on the graph of replacing f(x) by f(x) + k, k f(x), f(kx), and f(x + k) for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology.
F-TF.1 Understand radian measure of an angle as the length of the arc on the unit circle subtended by the angle.
F-TF.2 Explain how the unit circle in the coordinate plane enables the extension of trigonometric functions to all real numbers, interpreted as radian measures of angles traversed counterclockwise around the unit circle.
F-TF.3 Use special triangles to determine geometrically the values of sine, cosine, tangent for pi/3, pi/4 and pi/6, and use the unit circle to express the values of sine, cosine, and tangent for pi–x, pi+x, and 2pi–x in terms of their values for x, where x is any real number.
F-TF.4 Use the unit circle to explain symmetry (odd and even) and periodicity of trigonometric functions.
F-TF.5 Choose trigonometric functions to model periodic phenomena with specified amplitude, frequency, and midline.
F-TF.6 Understand that restricting a trigonometric function to a domain on which it is always increasing or always decreasing allows its inverse to be constructed.
F-TF.7 Use inverse functions to solve trigonometric equations that arise in modeling contexts; evaluate the solutions using technology, and interpret them in terms of the context.
G-SRT.6 Understand that by similarity, side ratios in right triangles are properties of the angles in the triangle, leading to definitions of trigonometric ratios for acute angles.
G-SRT.7 Explain and use the relationship between the sine and cosine of complementary angles.
G-SRT.8 Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems.
G-SRT.10 Prove the Laws of Sines and Cosines and use them to solve problems.
G-SRT.11 Understand and apply the Law of Sines and the Law of Cosines to find unknown measurements in right and non-right triangles (e.g., surveying problems, resultant forces).
G-C.5 Derive using similarity the fact that the length of the arc intercepted by an angle is proportional to the radius, and define the radian measure of the angle as the constant of proportionality; derive the formula for the area of a sector.

Understandings	**Essential Questions**	
Overarching Understanding	**Overarching**	**Topical**
• Solve right triangles using trigonometric and inverse trigonometric functions. • Convert between degrees and radians. • Solve real-world problems using trigonometric functions. • Graph trigonometric functions and their inverses. • Solve oblique triangles and find their area using various laws and formulas.	• What is analytical trigonometry? • Why do the shape of the sine and cosine curves look very similar? • What is radian measure? • How are trigonometric functions graphed on the	• When we do analytical trigonometry, why is radian measure used rather than degrees? • On the coordinate plane, when are the trigonometric functions positive? • How are the sine and cosine graphs similar?

*Optional

Related Misconceptions	
• The equations for finding arc length and area of a sector are the same when the angle measure is given in radians and degrees. • Quadrants are labeled in clockwise order. • All graphs of trigonometric functions are the same. • The Law of Sines and Law of Cosines only work for right triangles.	• What are the purposes of inverse trigonometric functions? • What parts of the triangle must be given to use either the Law of Sines or Cosines?

Knowledge Students will know…	**Skills** Students will be able to…
• The three trigonometric functions have inverse functions. • Radian measure is used for finding length in a linear measure. • You can use the coordinate plane to find the value of the ratio of an angle. • The coordinates of the unit circle are used to graph the basic functions. • The Laws of Sines and Cosines are used to find the missing parts of a non-right triangle.	• Formulate the six trigonometric ratios for 30°, 45° and 60°. • Rewrite degree measure to radian measure. • Construct and analyze the graph of trigonometric functions. • Estimate the measure of an angle given the ratio of the sides. • Calculate the missing parts of a triangle utilizing either the Law of Sines or Law of Cosines.

Assessment Evidence (Stage 2)

Performance Task Description

• **Goal** • **Role** • **Audience** • **Situation** • **Product/Performance** • **Standards**	**Related Rates (G-SRT.6)** https://drive.google.com/a/aulcs.org/file/d/0B29PaIp4pIr2Vy10em9MUHFFU0U/view?usp=sharing

Other Evidence

- Homework given after each class
- Practice questions showing understanding of concept (both individually and in groups).
  - Using ideas like think-pair-share
- Quizzes administered after each section is completed to show knowledge gained.
- Review activity to show if students are prepared for unit test.
- Test administered at the end of the unit to show knowledge gained within the unit

*Optional

Learning Plan (Stage 3)

*Optional

• **Where are your students headed? Where have they been? How will you make sure the students know where they are going?**
• **How will you hook students at the beginning of the unit?**
• **What events will help students experience and explore the big idea and questions in the unit? How will you equip them with needed skills and knowledge?**
• **How will you cause students to reflect and rethink? How will you guide them in rehearsing, revising, and refining their work?**
• **How will you help students to exhibit and self-evaluate their growing skills, knowledge, and understanding throughout the unit?**
• **How will you tailor and otherwise personalize the learning plan to optimize the engagement and effectiveness of ALL students, without compromising the goals of the unit?**
• **How will you organize and sequence the learning activities to optimize the engagement and achievement of ALL students?**
	Students will work in small groups to assist each other with the questions for show understanding. Afterwards, they will branch out to the rest of their classmates for further clarification. Groups can be determined by the students or teacher.
	
	Activities during lesson will be differentiated by student ability. For example, each group will be assigned different questions on the same worksheet. If all students are working on the same level, this idea will be used and students will share their responses to the questions they were not asked to do.
	
	While students are getting a chance to work outside their small group, students will get the opportunity to explain how they derived their solution(s). The other group members will analyze and determine if the student's work has errors or not.
	
	Exit tickets will be completed the last 5-10 minutes of each lesson to show full understanding of each concept. Within each exit ticket, students will self-assess themselves by rating how well they felt they understand what was taught (1-4 scale).
	
	For review, students will "find matching triples". Each group of students will be given 30 different cards where they will either find the ratio or find the measure of the angle. They will match up the cards that have the same solution to form triples.
	
	Sequence:
	• Right Triangle Trigonometry
	  ○ Find values of trigonometric functions for acute angles of right triangles
	  ○ Solve right triangles
	• Degrees and Radians
	  ○ Convert degree measure of angles to radian measures, and vice versa.
	  ○ Use angle measures to solve real-world problems.
	• Trigonometric Functions on the Unit Circle
	  ○ Find values of trigonometric functions of any angle.
	  ○ Find values of trigonometric functions using the unit circle.
	• Graphing Sine and Cosine Functions
	  ○ Graph transformations of the sine and cosine functions.
	  ○ Use sinusoidal functions to solve problems.
	• Graphing Other Trigonometric Functions
	  ○ Graph tangent and reciprocal trigonometric functions.
	  ○ Graph damped trigonometric functions.
	• Inverse Trigonometric Functions
	  ○ Evaluate and graph inverse trigonometric functions.
	  ○ Find compositions of trigonometric functions.
	• The Law of Sines and the Law of Cosines
	  ○ Solve oblique triangles by using the Law of Sines or the Law of Cosines.

*Optional

*Optional

Title of Unit	Systems of Equations and Matrices	Grade Level	10, 11, 12
Curriculum Area	PreCalculus	Time Frame	3-4 Weeks
Developed By	Douglas Frost		

Identify Desired Results (Stage 1)

Content Standards

N-VM.8 Add, subtract, and multiply matrices of appropriate dimensions.
N-VM.9 Understand that, unlike multiplication of numbers, matrix multiplication for square matrices is not a commutative operation, but still satisfies the associative and distributive properties.
N-VM.10 Understand that the zero and identity matrices play a role in matrix addition and multiplication similar to the role of 0 and 1 in the real numbers. The determinant of a square matrix is nonzero if and only if the matrix has a multiplicative inverse.
N-VM.11 Multiply a vector (regarded as a matrix with one column) by a matrix of suitable dimensions to produce another vector. Work with matrices as transformations of vectors.
A-REI.8 Represent a system of linear equations as a single matrix equation in a vector variable.
A-REI.9 Find the inverse of a matrix if it exists and use it to solve systems of linear equations (using technology for matrices of dimension 3 × 3 or greater).
A-REI.12 Graph the solutions to a linear inequality in two variables as a half-plane (excluding the boundary in the case of a strict inequality), and graph the solution set to a system of linear inequalities in two variables as the intersection of the corresponding half-planes.

Understandings

Overarching Understanding

- Solve systems of linear equations using matrices and Gaussian or Gauss-Jordan elimination.
- Multiply matrices.
- Find determinants and inverses of 2 X 2 and 3 X 3 matrices.
- Solve systems of linear equations using inverse matrices and Cramer's Rule.
- Write partial fraction decompositions of rational expressions with linear and irreducible quadratic factors.
- Use linear programming to solve applications.
- Recognize situations in which there are no solutions or more than one solution of a linear programming application.

Related Misconceptions

Essential Questions

Overarching

- What are the ways to solve systems of equations ?
- What are determinants and their purpose?
- What are the purposes of partial fractions?
- Why do we use linear programming to find solutions of real-world situations?

Topical

- How do you use Gaussian and Gauss-Jordan elimination to solve systems of linear equations?
- What methods are used to find the determinant and inverse of a matrix?
- How can systems of equations be solved using inverse matrices and Cramer's Rule?

*Optional

• If the last row of a matrix is all zeros, there are no solutions to the systems of equations. • Multiply both sides of a matrix equation by the actual matrix to solve for x and y. • The solution set of linear programming is the intersection(s) of the graphs only.	
Knowledge Students will know…	**Skills** Students will be able to…
• The different methods used to solve systems of linear equations. • Perform operations with matrices. • Utilize partial fraction decomposition. • Linear programming is used to solve applications involving systems of linear inequalities.	• Synthesize the solutions of systems of linear equations. • Formulate new matrices using row operations. • Rewrite rational expressions using partial fraction decomposition. • Describe the solution set of systems of linear inequalities.
Assessment Evidence (Stage 2)	
Performance Task Description	
• **Goal** • **Role** • **Audience** • **Situation** • **Product/Performance** • **Standards**	**Nonlinear Optimization (A-REI.12)** https://drive.google.com/a/aulcs.org/file/d/0B29PaIp4pIr2MllrVXJRcy1XbXc/view?usp=sharing
Other Evidence	
• Homework given after each class • Practice questions showing understanding of concept (both individually and in groups). ○ Using ideas like think-pair-share • Quizzes administered after each section is completed to show knowledge gained. • Review activity to show if students are prepared for unit test. • Test administered at the end of the unit to show knowledge gained within the unit	
Learning Plan (Stage 3)	

*Optional

• **Where are your students headed? Where have they been? How will you make sure the students know where they are going?** • **How will you hook students at the beginning of the unit?** • **What events will help students experience and explore the big idea and questions in the unit? How will you equip them with needed skills and knowledge?** • **How will you cause students to reflect and rethink? How will you guide them in rehearsing, revising, and refining their work?** • **How will you help students to exhibit and self-evaluate their growing skills, knowledge, and understanding throughout the unit?** • **How will you tailor and otherwise personalize the learning plan to optimize the engagement and effectiveness of ALL students, without compromising the goals of the unit?** • **How will you organize and sequence the learning activities to optimize the engagement and achievement of ALL students?**	Students will work in small groups to assist each other with the questions for show understanding. Afterwards, they will branch out to the rest of their classmates for further clarification. Groups can be determined by the students or teacher. Activities during lesson will be differentiated by student ability. For example, each group will be assigned different questions on the same worksheet. If all students are working on the same level, this idea will be used and students will share their responses to the questions they were not asked to do. While students are getting a chance to work outside their small group, students will get the opportunity to explain how they derived their solution(s). The other group members will analyze and determine if the student's work has errors or not. Exit tickets will be completed the last 5-10 minutes of each lesson to show full understanding of each concept. Within each exit ticket, students will self-assess themselves by rating how well they felt they understand what was taught (1-4 scale). For review, students will reverse engineer systems of linear equations. They will create their own questions and be asked to solve them to make sure their solutions are correct. Sequence: • .Multivariable Linear Systems and Row Operations ○ Solve systems of equations using matrices and Gaussian or Gauss-Jordan elimination. • Matrix Multiplication, Inverses, and Determinants ○ Multiply matrices ○ Find determinants and inverses of 2 X 2 and 3 X 3 matrices. • Solving Linear Systems using Inverses and Cramer's Rule ○ Solve systems of linear equations using inverse matrices or Cramer's Rule. • Partial Fractions ○ Write partial fraction decompositions of rational expressions with linear factors or prime quadratic factors in the denominator. • Linear Optimization ○ Use linear programming to solve applications. ○ Recognize situations in which there are no solutions or more than one solution of a linear programming application.

*Optional

Title of Unit	Vectors	Grade Level	10, 11, 12
Curriculum Area	PreCalculus	Time Frame	3-4 Weeks
Developed By	Douglas Frost		

Identify Desired Results (Stage 1)

Content Standards

N-VM.1 Recognize vector quantities as having both magnitude and direction. Represent vector quantities by directed line segments, and use appropriate symbols for vectors and their magnitudes (e.g., v, |v|, ||v||, v).
N-VM.2 Find the components of a vector by subtracting the coordinates of an initial point from the coordinates of a terminal point.
N-VM.3 Solve problems involving velocity and other quantities that can be represented by vectors.
N-VM.4.a Add vectors end-to-end, component-wise, and by the parallelogram rule. Understand that the magnitude of a sum of two vectors is typically not the sum of the magnitudes.
N-VM.4.b Given two vectors in magnitude and direction form, determine the magnitude and direction of their sum.
N-VM.4.c Understand vector subtraction $v - w$ as $v + (-w)$, where $-w$ is the additive inverse of w, with the same magnitude as w and pointing in the opposite direction. Represent vector subtraction graphically by connecting the tips in the appropriate order, and perform vector subtraction component-wise.
N-VM.5.b Compute the magnitude of a scalar multiple cv using $||cv|| = |c|v$. Compute the direction of cv knowing that when $|c|v$ is not equal to 0, the direction of cv is either along v (for $c > 0$) or against v (for $c < 0$).
N-VM.10 Understand that the zero and identity matrices play a role in matrix addition and multiplication similar to the role of 0 and 1 in the real numbers. The determinant of a square matrix is nonzero if and only if the matrix has a multiplicative inverse.
N-VM.11 Multiply a vector (regarded as a matrix with one column) by a matrix of suitable dimensions to produce another vector. Work with matrices as transformations of vectors.

Understandings	Essential Questions	
Overarching Understanding	Overarching	Topical

*Optional

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- Represent and operate with vectors both geometrically and algebraically.
- Resolve vectors into their rectangular components.
- Write a vector as the linear combination of unit vectors.
- Find the dot product of two vectors and use the dot product to find the angle between them.
- Find the projection of one vector onto another.
- Graph and operate with vectors in space.
- Find the dot and cross products of vectors and angles between vectors in space.
- Find areas of parallelograms and volumes of parallelepipeds in space.

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- What are vectors and its purposes?
- How are vectors utilized in the coordinate plane?
- What are the purposes of dot products?
- What similarities are there with two-dimensional and three-dimensional vectors?
- How do you compute vectors?
- Why is it important to compute vectors?
- How do you find the distance and midpoint of a vector?
- How can you tell if two vectors are perpendicular?

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Related Misconceptions

- There is no similarity to vectors in Geometry or Physics.
- Three-dimensional vectors do not use the same methods of finding distance and midpoint as two-dimensional vectors.

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Knowledge
Students will know…

- A vector is a quantity that has both magnitude and direction.
- Vectors can be represented both geometrically and algebraically.
- Dot products can find the magnitude of a vector.
- Trigonometry with a combination of the dot product and magnitude can be used to find the angle formed by two vectors.
- When you are working with spacial coordinate planes, the coordinate system is split into octants.

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Skills
Students will be able to…

- Assemble the resultant of two vectors.
- Synthesize new vectors by performing operations on given vectors.
- Assess if two vectors are orthogonal.
- Formulate the angle between two vectors.

(All skills will be utilized in three-dimensional space as well)

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Assessment Evidence (Stage 2)

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Performance Task Description

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- **Goal**
- **Role**
- **Audience**
- **Situation**
- **Product/Performance**
- **Standards**

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<u>Vector Fields (All standards)</u>

https://drive.google.com/a/aulcs.org/file/d/0B29PaIp4pIr2SkM2ZDdGTWZ3dDA/view?usp=sharing

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*Optional

Other Evidence

- Homework given after each class
- Practice questions showing understanding of concept (both individually and in groups).
    - Using ideas like think-pair-share
- Quizzes administered after each section is completed to show knowledge gained.
- Review activity to show if students are prepared for unit test.
- Test administered at the end of the unit to show knowledge gained within the unit

Learning Plan (Stage 3)

*Optional

• **Where are your students headed? Where have they been? How will you make sure the students know where they are going?**
• **How will you hook students at the beginning of the unit?**
• **What events will help students experience and explore the big idea and questions in the unit? How will you equip them with needed skills and knowledge?**
• **How will you cause students to reflect and rethink? How will you guide them in rehearsing, revising, and refining their work?**
• **How will you help students to exhibit and self-evaluate their growing skills, knowledge, and understanding throughout the unit?**
• **How will you tailor and otherwise personalize the learning plan to optimize the engagement and effectiveness of ALL students, without compromising the goals of the unit?**
• **How will you organize and sequence the learning activities to optimize the engagement and achievement of ALL students?**
	Students will work in small groups to assist each other with the questions for show understanding. Afterwards, they will branch out to the rest of their classmates for further clarification. Groups can be determined by the students or teacher.
	
	Activities during lesson will be differentiated by student ability. For example, each group will be assigned different questions on the same worksheet. If all students are working on the same level, this idea will be used and students will share their responses to the questions they were not asked to do.
	
	While students are getting a chance to work outside their small group, students will get the opportunity to explain how they derived their solution(s). The other group members will analyze and determine if the student's work has errors or not.
	
	Exit tickets will be completed the last 5-10 minutes of each lesson to show full understanding of each concept. Within each exit ticket, students will self-assess themselves by rating how well they felt they understand what was taught (1-4 scale).
	
	For review, students will utilize vectors in Physics. They will be given real-world Physics situations where vectors are needed to compute solutions.
	
	Sequence:
	• .Introduction to Vectors
	  ○ Represent and operate with vectors geometrically.
	  ○ Solve vector problems and resolve vectors into their rectangular components.
	• Vectors in the Coordinate Plane
	  ○ Represent and operate with vectors in the coordinate plane.
	  ○ Write a vector as a linear combination of unit vectors.
	• Dot Products and Vector Projections
	  ○ Find the dot product of two vectors and use the dot product to find the angle between them.
	  ○ Find the projection of one vector onto another.
	• Vectors in Three-Dimensional Space
	  ○ Plot points and vectors in the three-dimensional coordinate system.
	  ○ Express algebraically and operate with vectors in space.
	• Dot and Cross Products of Vectors in Space
	  ○ Find dot products of and angles between vectors in space.
	  ○ Find cross products of vectors in space, and use cross products to find area and volume.

*Optional

*Optional

Title of Unit	Sequences and Series	Grade Level	10, 11, 12
Curriculum Area	PreCalculus	Time Frame	3-4 Weeks
Developed By	Douglas Frost		

Identify Desired Results (Stage 1)

Content Standards

A-SSE.4 Derive the formula for the sum of a finite geometric series (when the common ratio is not 1), and use the formula to solve problems.
A-APR.5 Know and apply the Binomial Theorem for the expansion of (x + y) to the n power in powers of x and y for a positive integer n, where x and y are any numbers, with coefficients determined for example by Pascal's Triangle.
F-IF.3 Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers.
F-BF.2 Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms.
F-LE.2 Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).
F-LE.3 Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically, or (more generally) as a polynomial function.

Understandings

Overarching Understanding

- Use sigma notation to represent and calculate sums of series.
- Find *n*th terms of arithmetic sequences and arithmetic series.
- Find *n*th terms of geometric sequences and geometric series.
- Use Pascal's Triangle or the Binomial Theorem to write binomial expansions.
- Use the Binomial Theorem to find the coefficients of specified terms in binomial expansions.
- *Use a power series to represent a rational function
- *Use power series representations to approximate values of transcendental functions.

Related Misconceptions

- Arithmetic and geometric sequences are the same.
- Permutations are used to find the coefficient of a binomial expansion.

Essential Questions

Overarching

- What are the uses of sigma notation?
- What are the differences between arithmetic and geometric sequences?
- How are combinations and Pascal's triangle related?
- *How accurate are power series when trying to find the value of rational and transcendental functions?

Topical

- How do you find the *n*th term of an arithmetic or geometric sequence?
- How do you compute arithmetic and geometric series?
- How do you use Pascal's Triangle to expand a binomial expression raised by a power?
- How do you find a specific term of a binomial expansion?
- *What methods are used to find the approximate value of a rational or transcendental function?

*Optional

Knowledge Students will know…	**Skills** Students will be able to…
• A sequences is an ordered lists of numbers. • A series is the sum of all the terms in a finite sequence or a partial sum of an infinite sequence. • A sequence is arithmetic when the difference between successive terms is a constant. • A sequence is geometric when the ratio between successive terms is a constant. • The Binomial Theorem is used to expand binomial expressions raised by a power. • *Power series are used to estimate the value of a rational or transcendental functions.	• Discriminate between arithmetic and geometric sequences. • Construct the formulas for arithmetic and geometric sequences. • Synthesize the sum of arithmetic and geometric series. • Construct expansions of binomial expressions raised by an exponent. • *Compose a power series to represent rational and transcendental functions.

Assessment Evidence (Stage 2)

Performance Task Description

• **Goal** • **Role** • **Audience** • **Situation** • **Product/Performance** • **Standards**	**Riemann Sum (A-SSE.4)** https://drive.google.com/a/aulcs.org/file/d/0B29PaIp4pIr2MDZwS3hVTVI2SXc/view?usp=sharing

Other Evidence

- Homework given after each class
- Practice questions showing understanding of concept (both individually and in groups).
    - Using ideas like think-pair-share
- Quizzes administered after each section is completed to show knowledge gained.
- Review activity to show if students are prepared for unit test.
- Test administered at the end of the unit to show knowledge gained within the unit

Learning Plan (Stage 3)

*Optional

• **Where are your students headed? Where have they been? How will you make sure the students know where they are going?** • **How will you hook students at the beginning of the unit?** • **What events will help students experience and explore the big idea and questions in the unit? How will you equip them with needed skills and knowledge?** • **How will you cause students to reflect and rethink? How will you guide them in rehearsing, revising, and refining their work?** • **How will you help students to exhibit and self-evaluate their growing skills, knowledge, and understanding throughout the unit?** • **How will you tailor and otherwise personalize the learning plan to optimize the engagement and effectiveness of ALL students, without compromising the goals of the unit?** • **How will you organize and sequence the learning activities to optimize the engagement and achievement of ALL students?**	Students will work in small groups to assist each other with the questions for show understanding. Afterwards, they will branch out to the rest of their classmates for further clarification. Groups can be determined by the students or teacher. Activities during lesson will be differentiated by student ability. For example, each group will be assigned different questions on the same worksheet. If all students are working on the same level, this idea will be used and students will share their responses to the questions they were not asked to do. While students are getting a chance to work outside their small group, students will get the opportunity to explain how they derived their solution(s). The other group members will analyze and determine if the student's work has errors or not. Exit tickets will be completed the last 5-10 minutes of each lesson to show full understanding of each concept. Within each exit ticket, students will self-assess themselves by rating how well they felt they understand what was taught (1-4 scale). For review, students will "sort through the laundry". They will be given different series and sort through them to determine if a series is arithmetic or geometric. Then they will construct the expression of each series and find the given term within it. Sequence: • .Sequences, Series, and Sigma Notation ○ Investigate several different types of sequences. ○ Use sigma notation to represent and calculate sums of series. • Arithmetic Sequences and Series ○ Find *n*th terms of arithmetic means of arithmetic sequences. ○ Find sums of *n* terms of arithmetic series. • Geometric Sequences and Series ○ Find *n*th terms of geometric means of arithmetic sequences. ○ Find sums of *n* terms of geometric series. ○ Find sums of infinite geometric series. • The Binomial Theorem ○ Use Pascal's Triangle to write binomial expressions. ○ Use the Binomial Theorem to write and find the coefficients of specified terms in binomial expressions. • *Functions as Infinite Series ○ Use a power series to represent a rational function. ○ Use power series representations to approximate values of transcendental functions.

*Optional

*Optional

Title of Unit	*Conic Sections and Parametric Equations	Grade Level	10, 11, 12
Curriculum Area	PreCalculus	Time Frame	4-5 Weeks
Developed By	Douglas Frost		

Identify Desired Results (Stage 1)

Content Standards

G-GPE.1 Derive the equation of a circle of given center and radius using the Pythagorean Theorem; complete the square to find the center and radius of a circle given by an equation.
G-GPE.2 Derive the equation of a parabola given a focus and directrix.
G-GPE.3 Derive the equations of ellipses and hyperbolas given the foci, using the fact that the sum or difference of distances from the foci is constant.
G-GMD.4 Identify the shapes of two-dimensional cross-sections of three-dimensional objects, and identify three-dimensional objects generated by rotations of two-dimensional objects.

Understandings

Overarching Understanding

- Analyze, write, and graph equations of parabolas, ellipses, circles, and hyperbolas.
- Use equations to identify types of conic sections.
- Use rotation of axes to write equations of rotated conic sections.
- Graph rotated conic sections.
- Graph parametric equations.
- Solve problems related to the motion of projectiles.

Related Misconceptions

- The equations of parabolas using the focus is the same as using the vertex.
- The equations for ellipses and hyperbolas are the same.

Essential Questions

Overarching

- What other ways is it possible to write the equations of parabolas?
- What are the differences from graphing ellipses and hyperbolas?
- How are equations of rotations of conic sections written?
- What are parametric equations?

Topical

- How is the focus and directrix used to find the equation of any parabola?
- What key information is necessary to find the equation of ellipses, circles, and hyperbolas?

Knowledge
Students will know…

Skills
Students will be able to…

*Optional

• There is a way to find the equation of a parabola whose axis of symmetry is horizontal.
• Equations for ellipses and hyperbolas depend on where the vertices are located.
• Rotations of conic sections are similar to shapes of parabolas, ellipses, and hyperbolas.
• Parametric equations can illustrate the position of an object both horizontally and vertically as a function of time.
	• Construct equations for the following types of graphs:
	  ○ Parabolas
	  ○ Ellipses
	  ○ Circles
	  ○ Hyperbolas
	  ○ Rotations of conic sections
	  ○ Parametric
	• Summarize the properties of each graph.
	• Synthesize solutions to real-world situations utilizing the graphs of each of the functions.

Assessment Evidence (Stage 2)

Performance Task Description

• **Goal** • **Role** • **Audience** • **Situation** • **Product/Performance** • **Standards**	**Solids of Revolution (G-GMD.4)** https://drive.google.com/a/aulcs.org/file/d/0B29PaIp4pIr2cldxY3NTMW9Ma2c/view?usp=sharing

Other Evidence

- Homework given after each class
- Practice questions showing understanding of concept (both individually and in groups).
  - Using ideas like think-pair-share
- Quizzes administered after each section is completed to show knowledge gained.
- Review activity to show if students are prepared for unit test.
- Test administered at the end of the unit to show knowledge gained within the unit

Learning Plan (Stage 3)

*Optional

• **Where are your students headed? Where have they been? How will you make sure the students know where they are going?**
• **How will you hook students at the beginning of the unit?**
• **What events will help students experience and explore the big idea and questions in the unit? How will you equip them with needed skills and knowledge?**
• **How will you cause students to reflect and rethink? How will you guide them in rehearsing, revising, and refining their work?**
• **How will you help students to exhibit and self-evaluate their growing skills, knowledge, and understanding throughout the unit?**
• **How will you tailor and otherwise personalize the learning plan to optimize the engagement and effectiveness of ALL students, without compromising the goals of the unit?**
• **How will you organize and sequence the learning activities to optimize the engagement and achievement of ALL students?**
	Students will work in small groups to assist each other with the questions for show understanding. Afterwards, they will branch out to the rest of their classmates for further clarification. Groups can be determined by the students or teacher.
	
	Activities during lesson will be differentiated by student ability. For example, each group will be assigned different questions on the same worksheet. If all students are working on the same level, this idea will be used and students will share their responses to the questions they were not asked to do.
	
	While students are getting a chance to work outside their small group, students will get the opportunity to explain how they derived their solution(s). The other group members will analyze and determine if the student's work has errors or not.
	
	Exit tickets will be completed the last 5-10 minutes of each lesson to show full understanding of each concept. Within each exit ticket, students will self-assess themselves by rating how well they felt they understand what was taught (1-4 scale).
	
	For review, students will "match 'em up". They will be given the graph and match it with its equation. In addition, they will need to summarize the equation with its properties.
	
	Sequence:
	• .Parabolas
	  ○ Analyze and graph equations of parabolas.
	  ○ Write equations of parabolas.
	• Ellipses and Circles
	  ○ Analyze and graph equations of ellipses and circles.
	  ○ Use equations to identify ellipses and circles.
	• Hyperbolas
	  ○ Analyze and graph equations of hyperbolas.
	  ○ Use equations to identify types of conic sections.
	• Rotations of Conic Sections
	  ○ Find rotation of axes to write equations of rotated conic sections.
	  ○ Graph rotated conic sections.
	• Parametric Equations
	  ○ Graph parametric equations.
	  ○ Solve problems related to the motion of properties.

*Optional

Title of Unit	Introduction to Calculus	Grade Level	10, 11, 12
Curriculum Area	PreCalculus	Time Frame	4-6 Weeks
Developed By	Douglas Frost		

Identify Desired Results (Stage 1)

Content Standards

No common core standards related to this unit

Understandings

Overarching Understanding

- Estimate limits of functions at fixed values and at infinity.
- Evaluate limits of polynomials and rational functions at selected points and at infinity.
- Find instantaneous rates of change by calculating slopes of tangent lines.
- Find instantaneous rates of change by calculating derivatives.
- Use the Product and Quotient Rules to calculate derivatives.
- Approximate the areas under a curve using rectangles.
- Approximate the area under a curve using definite integrals and integration.
- Find antiderivatives.
- Use the Fundamental Theorem of Calculus

Related Misconceptions

- All functions have a finite limit.
- The tangent equation is the derivative function.
- The antiderivative and integral are not related.

Essential Questions

Overarching

- How are limits calculated graphically and algebraically?
- What ways is calculus used to compute velocity of an object?
- What are the purposes to find the area under a curve?
- Why is integration considered to be the antiderivative?

Topical

- What is limit notation?
- How are limits founds as they are approaching infinity?
- How are tangent lines used to determine instantaneous velocity?
- What rules are utilized to find the derivative of a given function?
- What is the Fundamental Theorem of Calculus and its role?

Knowledge

Students will know…

Skills

Students will be able to…

*Optional

• Limits are used to show where a function will approach at a certain value. • The Instant Rate of Change Formula is used to find the slope of a curve at a certain point. • Derivatives are an easier way to find the slope of a curve at a point. • The integral of a function is the antiderivative, which is performing the reverse derivative of a given function. • To find the exact area under a curve, you can use the Fundamental Theorem of Calculus to do so.	• Compare the limits of a function graphically and algebraically. • Use the Instantaneous Rate of Change Formula to construct the derivative of a function. • Compare and contrast the area of a curve when using different size rectangles. • Construct the integral of a function to find the exact area of a curve.

Assessment Evidence (Stage 2)

Performance Task Description

• **Goal** • **Role** • **Audience** • **Situation** • **Product/Performance** • **Standards**	**The Chain Rule (no standard)** https://drive.google.com/a/aulcs.org/file/d/0B29PaIp4pIr2b3doa1d5cDJCOEE/view?usp=sharing

Other Evidence

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Learning Plan (Stage 3)

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	Exit tickets will be completed the last 5-10 minutes of each lesson to show full understanding of each concept. Within each exit ticket, students will self-assess themselves by rating how well they felt they understand what was taught (1-4 scale).
	
	For review, students will be given a function and be asked to differentiate and integrate the function.
	
	Sequence:
	• Estimating Limits Graphically
	  ○ Estimates limits of functions at fixed values.
	  ○ Estimate limits of functions at infinity.
	• Evaluating Limits Algebraically
	  ○ Evaluate limits of polynomial and rational functions at selected points.
	  ○ Evaluate limits of polynomial and rational functions at infinity.
	• Tangent Lines and Velocity
	  ○ Find instantaneous rates of change by calculating slopes of tangent lines.
	  ○ Find average and instantaneous velocity.
	• Derivatives
	  ○ Find instantaneous rates of change by calculating derivatives.
	  ○ Use the Product and Quotient Rules to calculate derivatives.
	• Area Under a Curve and Integration
	  ○ Approximate the area under a curve using rectangles.
	  ○ Approximate the area under a curve using definite integrals and integration.
	• The Fundamental Theorem of Calculus
	  ○ Find antiderivatives
	  ○ Use the Fundamental Theorem of Calculus

*Optional