Unit 1	Relationships between Quantities and Reasoning with Equations	**Grade Level**	Grade 8 Level 3
Curriculum Area	Mathematics	**Time Frame**	3-4 weeks
Developed By	Munira Jamali		

Identify Desired Results (Stage 1)

Content Standards

Algebra - Seeing Structure in Expressions
A.SSE.1 Interpret expressions that represent a quantity in terms of its context.
★ a. Interpret parts of an expression, such as terms, factors, and coefficients.
b. Interpret complicated expressions by viewing one or more of their parts as single entity. For example, interpret P(1+r)n as the product of P and a factor not depending on P.
Algebra - Reasoning with Equations and Inequalities
A.REI.1 Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method.
A.REI.3 Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.
A.REI.3.1 Solve one-variable equations and inequalities involving absolute value, graphing the solutions and interpreting them in context.
Numbers - Quantities
N.Q.1 Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays.
N.Q.2 Define appropriate quantities for the purpose of descriptive modeling.
N.Q.3 Choose a level of accuracy appropriate to limitations on measurement when reporting quantities.
Algebra - Creating Equations
A.CED.1 Create equations and inequalities in one variable and use them to solve problems. Include equations arising from linear and quadratic functions, and simple rational and exponential functions.
A.CED.2 Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.
A.CED.3 Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or non-viable options in a modeling context. For example, represent inequalities describing nutritional and cost constraints on combinations of different foods.
A.CED.4 Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations. For example, rearrange Ohm's law V = IR to highlight resistance R.

Understandings	Essential Questions	
Overarching Understanding	**Overarching**	**Topical**

Understand that numbers in real world applications often have units attached to them, and they are considered quantities. • Understand the structure of algebraic expressions and polynomials. • Understand general linear equations ($y=mx+b$, $m≠0$) and their graphs and extend this to work with absolute value equations, linear inequalities, and systems of linear equations. • Use properties of equality and order of operation to solve an equation by using inverse operations. • Solve equations and inequalities give all the values of a variable that make the equation/inequality true. • The values that define inequalities are graphically represented by either: a set of linear values or the areas represented above or below the linear values.	What are the "pieces" of an algebraic expression? What do they represent in the context of the real-world situation? • What do the parts of an expression tell us in a real-world context? • How would you describe the difference between an expression and an equation? • How do the properties of equality and order of operations extend to support the solving of an equation? • Why is it important to be able to solve linear equations and inequalities in one variable?	**When and how is mathematics used in solving real world problems** How are equations and inequalities used to solve real world problems? **What characteristics of problems would determine how to model the situation and develop a problem solving strategy?** What characteristics of problems would help to distinguish whether the situation could be modeled by a linear or an exponential model?
Differentiation		

UDL/FRONT LOADING Prerequisites: Familiarity with g order of operations, exponents, variables, coefficients, functions, domain, quadrants, x-axis, y-axis, line, fractions, integers, equations, rational numbers, irrational numbers, real numbers, expressions by utilizing sentence stems, language frames, visuals, and cloze reading Experience in problem solving, reading and communicating, estimating and verifying answers and solutions, logical reasoning, and using technology. Students must be able to use the language of mathematics orally and in writing to explain the thinking processes, mathematical concepts and solution strategies they use in solving problems. Students, at least informally, should become familiar with examples of inductive and deductive reasoning. Students should become proficient in the use of scientific calculators and graphing calculators to enhance their understanding of mathematical ideas and concepts. **<u>Acceleration</u>** Due to their intuitive understanding of mathematical function and processes, students who are mathematically gifted may skip over steps and be unable to explain how they arrived at the correct answer to a problem. Utilize Math Practice 3 with them often. Provide students with opportunities to share their previous knowledge and avoid redundant learning by being encouraged to learn the sophisticated and advanced information and skills of the curriculum or related curriculums at their own rate. This also includes the opportunity for students to make personal meaning of the lesson. Provide students with a variety of learning/ assessment options. Use engaging, active, and grounded in reality activities. The increased complexity of the problems should require higher order thinking skills and provide opportunities for open-ended responses. Students who are accelerated in mathematics often demonstrate an uneven pattern of mathematical understanding and development, and may be much stronger in concept development than they are in	• How do you graphically represent the solutions to a linear equation? • How do you graphically represent the values that define linear inequalities?	When is it advantageous to represent relationships between quantities symbolically? numerically? graphically? **Why is it necessary to follow set rules/ procedures/properties when manipulating numeric or algebraic expressions?** How can, the structure of expressions/equations/ inequalities, units of measure used, and mathematical properties help determine a solution strategy?

Knowledge Students will know…	**Skills** Students will be able to…
• Interpret the structure of expressions • Understand solving equations as a process of reasoning and explain the reasoning. • Solve equations and inequalities in one variable. • Reason quantitatively and use units to solve problems.	To analyze and explain the process of solving an equation. Develop fluency writing, interpreting, and translating between various forms of linear equations and inequalities, and using them to solve problems Master the solution of linear equations and apply related solution techniques and the laws of exponents to the creation and solution of simple exponential equations. All of this work is grounded on understanding quantities and on relationships between them.

Assessment Evidence (Stage 2)

Performance Task Description

• **Goal** • **Role** • **Audience** • **Situation** • **Product/ Performance** • **Standards**	**Solving Equations in One Variable: (8.EE)** http://map.mathshell.org/materials/lessons.php?taskid=442 Goals Students are able to: Solve linear equations in one variable with rational number coefficients. • Collect like terms. • Expand expressions using the distributive property. • Categorize linear equations in one variable as having one, none, or infinitely many solutions. It also aims to encourage discussion on some common misconceptions about algebra. **8.EE: Analyze and solve linear equations and pairs of simultaneous linear equations.** **Sorting Equations and Identities** http://map.mathshell.org/materials/lessons.php?taskid=218&subpage=concept Goals • Recognize the differences between equations and identities. • Substitute numbers into algebraic statements in order to test their validity in special cases. • Resist common errors when manipulating expressions such as $2(x - 3) = 2x - 3$; $(x + 3)^2 = x^2 + 3^2$. • Carry out correct algebraic manipulations. It also aims to encourage discussion on some common misconceptions about algebra A-REI: Solve equations and inequalities in one variable.

Other Evidence

Manipulating Polynomials:
http://map.mathshell.org/materials/lessons.php?taskid=437&subpage=concept
Defining Regions of Inequalities:
http://map.mathshell.org/materials/lessons.php?taskid=219&subpage=concept
Interpreting Algebraic Expressions:
http://map.mathshell.org/materials/lessons.php?taskid=219&subpage=concept

Learning Plan (Stage 3)

• **Where are your students headed? Where have they been? How will you make sure the students know where they are going?** • **How will you hook students at the beginning of the unit?** • **What events will help students experience and explore the big idea and questions in the unit? How will you equip them with needed skills and knowledge?** • **How will you cause students to reflect and rethink? How will you guide them in rehearsing, revising, and refining their work?** • **How will you help students to exhibit and self-evaluate their growing skills, knowledge, and understanding throughout the unit?** • **How will you tailor and otherwise personalize the learning plan to optimize the engagement and effectiveness of ALL students, without compromising the goals of the unit?** • **How will you organize and sequence the learning activities to optimize the engagement and achievement of ALL students?**	Start by directing students to understand written sequence of steps for solving linear equations which is the code for a narrative line of reasoning that would use words like "if", "then", "for all" and "there exists." In the process of learning to solve equations, students should learn certain "if - then" moves: e.g. "if $x = y$ then $x + c = y + c$ for any c." The first requirement in this domain (REI) is that students understand that solving equations is a process of reasoning (A.REI.1). Have students reason through problems with careful selection of units, and how to use units to understand problems and make sense of the answers they deduce. Example As Felicia gets on the freeway to drive to her cousin's house, she notices that she is a little low on gas. There is a gas station at the exit she normally takes, and she wonders if she will have to get gas before then. She normally sets her cruise control at the speed limit of 70 mph and the freeway portion of the drive takes about an hour and 15 minutes. Her car gets about 30 miles per gallon on the freeway, and gas costs $3.50 per gallon. a. Describe an estimate that Felicia might do in her head while driving to decide how many gallons of gas she needs to make it to the gas station at the other end. b. Assuming she makes it, how much does Felicia spend per mile on the freeway? Students will create multiple ways to rewrite an expression that represents its equivalent form. http://a4a.learnport.org/page/algebra-tiles The use of algebraic tiles to establish a visual understanding of algebraic expression and the meaning of terms, factors, and coefficients.

Unit 2	Linear Relationships	**Grade: Level 3**		
Curriculum Area	Mathematics	**Time Frame**	2-3 weeks	
Developed By	Munira Jamali			
Identify Desired Results (Stage 1)				
Content Standards				

Algebra - Reasoning with Equations and Inequalities
A.REI.10. Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line).
A.REI.11. Explain why the x-coordinates of the points where the graphs of the equations y=f(x) and y=g(x) intersect are the solutions of the equation f(x) = g(x) find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where f(x) and/ or g(x) are linear, polynomial, rational, absolute value, exponential, and logarithmic functions. ★
A.REI.12. Graph the solutions to a linear inequality in two variables as a half-plane (excluding the boundary in the case of a strict inequality), and graph the solution set to a system of linear inequalities in two variables as the intersection of the corresponding half-planes.
A-REI.5. Prove that, given a system of two equations in two variables, replacing one equation by the sum of that equation and a multiple of the other produces a system with the same solutions.
A-REI.6. Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables
A.CED.3 Represent constraints by equations or inequalities, and by the systems of equations and/or inequalities, and interpret solutions as viable or non-viable options in a modeling content. For example, represent inequalities describing nutritional and cost constraints on combinations of different foods.
Functions - Interpreting Functions

F.IF.1. Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If f is a function and x is an element of its domain, then f(x) denotes the output of f corresponding to the input x. The graph of f is the graph of the equation y = f(x).
F.IF.2. Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.
F.IF.3. Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers. For example, the Fibonacci sequence is defined recursively by $f(0) = f(1) = 1$, $f(n + 1) = f(n) + f(n - 1)$ for $n \geq 1$.
F.IF.7. Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. a. Graph linear and quadratic functions and show intercepts, maxima, and minima. b. Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions. ★
F.IF.9. Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). For example, given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum

Understandings	**Essential Questions**	
Overarching Understanding	**Overarching**	**Topical**

Apply rules so that polynomials form a system analogous to integers. Write in equivalent forms that represent both linear and exponential functions and construct functions to describe the situation and to find solutions Apply rules that builds a function that models a relationship between two quantities Represent equations and inequalities in one variable in various ways and use them to extend the properties of exponents to rational exponents Understand the relationship between quantities of two systems of equations and the methods to solve two system of linear equations Model with linear and exponential functions. Systems of equations compare at least two different functions Write in equivalent forms that represent both linear and exponential functions and construct functions to describe the situation and to find solutions Apply rules that builds a function that models a relationship between two quantities Represent equations and inequalities in one variable in various ways and use them to extend the properties of exponents to rational exponents Understand the relationship between quantities of two systems of equations and the methods to solve two system of linear equations	How will students identify the different parts of a two system equation and explain their meaning within the context of the problem? 2. What is the importance of identifying the structure of functions and using different ways to represent them? 3. Why is it important to identify and extend the properties of exponents to rational exponents? 4. When do students decide the best method to solve an inequality? 5. How do you know which method to use in solving a system of equations? 6. Why is it important to analyze functions using different representations? 7 How do I analyze algebraic equations/ inequalities to solve problems? 8. What must students understand in order to create equations that describe numbers or relationships? 9. How do students know the most efficient ways to build a function that models a relationship between two quantities?	What does the slope of a line indicate about the line? • What information does the equation of a line give you? • How are equations and graphs related?
DIFFERENTIATION		

UDL/FRONT LOADING **Prerequisites** • Students apply their understanding of the properties of exponents. • Students apply and extend their knowledge of rational numbers to exponents and to find the values of numerical values that include those numbers. • Students apply their knowledge about the meaning of the representation of radicals with rational exponents. • Students will understand that if the two sides of one equation are equal, and the two sides of another equation are equal, then the sum (or difference) of these is equal. • Students will extend their knowledge of learning the relationship between the algebraic representation and its graph. • Students will use their prior knowledge of creating tables of values for function to find a solutions. • Students will extend their prior knowledge of graphing two equations and be able to interpret the intersections of the graph as the solution to the original equation. **ACCELERATION** Students will design a word problem that reflects the use of graphing inequalities. • Students will write a real-life scenario and explain the process needed to solve a system of linear equations with two variables. • Student will create a real world problem where students will build a function that model a relationship between two quantities. • Students will explain the relationship of properties of exponents to exponential functions. • Students will compare and contrast the properties of a linear equation and linear inequality equation. • Students discuss the following question: Which quantity will grow more rapidly; one that is increasing exponentially, one that is increasing quadratically or one that is increasing linearly? **INTERVENTION** Use real-word context examples to demonstrate the meaning of the parts of a system of equations for the students	10. Why is it important to understand solving a system of linear and exponential relationships in two variables algebraically and graphically? 11. Is there functional relationship in non-linear and ambiguous data? 12. What is the difference in linear and exponential functions and how is that represented graphically? 13. What real-life situations would need exponential or linear function functions to describe them? 14. What is the relationship of a recursive function on the table and graph that represents it? 15. How might an arithmetic sequence be connected to a linear function? 16. How might a geometric sequence be connected to an exponential function??		

Knowledge Students will know…	**Skills** Students will be able to…
Extend the properties of exponents to rational exponents. Build a function that models a relationship between two quantities. Build new functions from existing functions Interpret functions that arise in applications in terms of a context. Solve systems of equations. Represent and solve equations and inequalities Graphically	Develop the concepts of domain and range in function notation. They move beyond viewing functions as processes that take inputs and yield outputs and start viewing functions as objects in their own right. Explore many examples of functions, including sequences; they interpret functions given graphically, numerically, symbolically, and verbally, translate between representations, and understand the limitations of various representations. Work with functions given by graphs and tables, keeping in mind that, depending upon the context, these representations are likely to be approximate and incomplete. Their work includes functions that can be described or approximated by formulas as well as those that cannot. When functions describe relationships between quantities arising from a context, students reason with the units in which those quantities are measured. Explore systems of equations and inequalities, and they find and interpret their solutions. Students build on and informally extend their understanding of integer exponents to consider exponential functions. They compare and contrast linear and exponential functions, distinguishing between additive and multiplicative change. Interpret arithmetic sequences as linear functions and geometric sequences as exponential functions
Assessment Evidence (Stage 2)	
Performance Task Description	

• **Goal** • **Role** • **Audience** • **Situation** • **Product/ Performance** • **Standards**	• Building and Solving Equations 2: A-REI http://map.mathshell.org/materials/lessons.php?taskid=554#task554 Goals Solving equations where the unknown appears once or more than once Solving equations in more than one way Standard A-REI: Understand solving equations as a process of reasoning and explain the reasoning. Solve equations and inequalities in one variable http://www.ccsstoolbox.com/parcc/PARCCPrototype_main.html Celllular Growth Goals Students write geometric sequences both recursively and with an explicit formula and use the two representations to model situations. Students consider the use of one or more representations to solve a real-world problem and choose the type of sequence to represent a situation. Rabbit Growth Goals student will use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another.

Other Evidence

College Board
http://achieve.lausd.net/cms/lib08/CA01000043/Centricity/domain/244/alignment%20doc/Algebra%201%20Textbook%20Aligment%20-%20SpringBoard.pdf
Engage New York
http://www.engageny.org/sites/default/files/resource/attac hments/algebra-i-m1-copy-ready-materials.pdf
Illustrative Mathematics
A Sum of Functions – F. BF. 1a
http://www.illustrativemathematics.org/illustrations/230

Learning Plan (Stage 3)

• **Where are your students headed? Where have they been? How will you make sure the students know where they are going?** • **How will you hook students at the beginning of the unit?** • **What events will help students experience and explore the big idea and questions in the unit? How will you equip them with needed skills and knowledge?** • **How will you cause students to reflect and rethink? How will you guide them in rehearsing, revising, and refining their work?** • **How will you help students to exhibit and self-evaluate their growing skills, knowledge, and understanding throughout the unit?** • **How will you tailor and otherwise personalize the learning plan to optimize the engagement and effectiveness of ALL students, without compromising the goals of the unit?** • **How will you organize and sequence the learning activities to optimize the engagement and achievement of ALL students?**	**Use Analogy in the Context of the Math Exponential Growth.** When a quantity grows with time by a multiplicative factor greater than 1, it is said the quantity grows exponentially. Hence, if an initial population of bacteria, $P0$, doubles each day, then after t days, the new population is given by $P(t)$ = = This expression can be generalized to include different growth rates, as in (t) = $P0r\ t$. The following example illustrates the type of problem that students can face after they have worked with basic exponential functions like these. Example. On June 1, a fast growing species of algae is accidentally introduced into a lake in a city park. It starts to grow and cover the surface of the lake in such a way that the area covered by the algae doubles every day. If it continues to grow unabated, the lake will be totally covered and the fish in the lake will suffocate. At the rate it is growing, this will happen on June 30. a. When will the lake be covered halfway? b. Write an equation that represents the percentage of the surface area of the lake that is covered in algae as a function of time (in days) that passes since the algae was introduced into the lake. Facilitate a discussion that would direct students to generate recursive formula for the sequence $P(n)$, which gives the population at a given time period n in terms of the population n-1 for the following example: Populations of bacteria can double every 6 hours under ideal conditions, at least until the nutrients in its supporting culture are depleted. This means a population of 500 such bacteria would grow to 1000, etc. Use of Exit Slips to assess student understanding. http://daretodifferentiate.wikispaces.com/PreAssessment EPR) strategies for whole group instruction. Strategies to check for understanding: Individual White Boards, Fist of Five, Exit Slip, etc. grow unabated, the lake will be totally covered and the fish in the lake will suffocate. At the rate it is growing, this will happen on June 30. a. When will the lake be covered halfway? b. Write an equation that represents the percentage of the surface area of the lake that is covered in algae as a function of time (in days) that passes since the algae was

Expressions and Equations	Expression and Equations	**Grade Level**	8th Grade Level3
Curriculum Area	Mathematics	**Time Frame**	4-5 weeks
Developed By	Munira Jamali		
Identify Desired Results (Stage 1)			
Content Standards			

A-SSE.1 Interpret expressions that represent a quantity in terms of its context.★ a. Interpret parts of an expression, such as terms, factors, and coefficients. b. Interpret complicated expressions by viewing one or more of their parts as a single entity. For example, interpret P(1+r)n as the product of P and a factor not depending on P. A-SSE.2 Use the structure of an expression to identify ways to rewrite it. For example, see $x^4 – y^4$ as $(x^2)^2 – (y^2)^2$, thus recognizing it as a difference of squares that can be factored as $(x^2 – y^2)(x^2 + y^2)$. A-SSE.3 Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.★ a. Factor a quadratic expression to reveal the zeros of the function it defines. b. Complete the square in a quadratic expression to reveal the maximum or minimum value of the function it defines. c. Use the properties of exponents to transform expressions for exponential functions. For example the expression 1.15t can be rewritten as $(1.15^{1/2})^{12t} \approx 1.012^{12t}$ to reveal the approximate equivalent monthly interest rate if the annual rate is 15%. A-APR.1 Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials. A-CED.1 Create equations and inequalities in one variable and use them to solve problems. Include equations arising from linear and quadratic functions, and simple rational and exponential functions. A-CED.2 Create equations in two or more variables to represent relationships graph equations on coordinate axes with labels and scales. A-CED.4 Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations. For example, rearrange Ohm's law V = IR to highlight resistance R. A-REI.4 Solve quadratic equations in one variable. a. Use the method of completing the square to transform any quadratic equation in x into an equation of the form (x – p) 2 = q that has the same solutions. Derive the quadratic formula from this form. b. Solve quadratic equations by inspection (e.g., for $x^2 = 49$), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as a ± bi for real numbers a and b

Understandings	**Essential Questions**	
Overarching Understanding	**Overarching**	**Topical**

Represent a quantity in terms of an expression, such as terms, factors, and coefficients by viewing one or more of their parts as a single entity. • Write in equivalent forms to find solutions that reveal and explain properties of quadratic expressions from completing the square, factoring, and using properties of exponents. • Apply rules so that polynomials form a system analogous to integers. • Represent equations and inequalities in one variable in various ways and use them to solve problems. • Understand the relationship between quantities of two or more variables through graphing on a coordinate plane system. • Transform quadratic equations using the method of completing the square to derive a solution. • Recognize the various methods to solve quadratic equations stemming from an initial form as appropriate: taking the square root, completing the square, using the quadratic formula, and factoring. • Identify when the quadratic formula gives complex solutions. • Solve systems of linear equations in two variables algebraically and graphically	1. How will students identify the different parts of an expression and explain their meaning within the context of the problem? 2. What is the importance of identifying the structure of an expression and ways to rewrite it? 3. Why is it important to solve and produce equivalent forms of an expression? 4. When is factoring the best method to solve a quadratic equation? 5. When is completing the square useful to reveal the maximum or minimum value of the function it defines? 6. How do students know which method to use in solving quadratic equations? 7. Why is it important to know the operations of integers to understand the properties of polynomials? 8. How can students analyze algebraic equations/inequalities to solve problems?	Why do we use equations to solve problems? Why do you perform operations on **both** sides of an equation? How is thinking algebraically different from thinking arithmetically? How do the properties contribute to algebraic understanding?
Differentiaiton		

UDL/FRONT LOADING Have students apply their understanding of expressions as sums of terms and products of factors to find and use the properties of operations to find the values of numerical expressions. • Engage students in a discussion regarding applying their prior knowledge about the order of operations and properties of operations to transform simple expressions. Transformations require an understanding of the rules for multiplying negative numbers, and properties of integer exponents. • Involve students to have a discussion that would have them extend their knowledge of analyzing and solving linear equations and pairs of simultaneous linear equations. Have them use their prior knowledge of graphing proportional relationships, lines, and linear equations to approaching system of linear and quadratic equations with two variables. **ACCELERATION** Provide the students with a problem (either quadratic equation or system of linear equations), ask them to solve it by different methods (for system: algebraic methods – elimination, substitution, addition, etc. and graphing; for quadratics – factoring, completing by square, quadratic formula, graphing), then have them write an explanation of which method was most relevant to the problem type. • Take students through the process of designing word problems involving quadratic equations and functions. Have students write a scenario and explain the process needed to solve a system of linear and quadratic equations with two variables. • Create a real world problem where factoring is the best method to solve a quadratic expression. Have students apply their math knowledge of quadratic equations to solve a word problem they have created. **INTERVENTION** Use of real context examples to demonstrate the meaning of quadratics equation, such rocket trajectory, basketball path when thrown to the hoop, etc. • Have students use technology, such as graphing calculator, graphing apps, and other software to graph both a linear function and quadratic function on the same plane. Engage them in a discussion to identify the point of intersection of the linear graph and the quadratics graph and discuss what that means. • Provide a situation that uses realia to further	solve problems? 9. What must students understand in order to create equations that describe numbers or relationships? 10. How do students know which is the most effective way to solve a quadratic equation ? 11. Why is it important to understand solving a system of linear and quadratic equations in two variables algebraically and graphically? 12. How are the methods of solving a quadratic equation related? 13. How do students know when the roots of a quadratic equation are real or complex? 14. Why are the methods of solving quadratic equations not learned in isolation?	

Knowledge Students will know…	**Skills** Students will be able to…
To interpret the structure of expressions To write expressions in equivalent forms to solve problems. Perform arithmetic operations on polynomials. Create equations that describe numbers or relationships. Solve equations and inequalities in one variable. Solve systems of equations.	Build on their knowledge from Unit 2, where they extended the laws of exponents to rational exponents. Apply this new understanding of numbers and strengthen their ability to see structure in and create quadratic and exponential expressions. Create and solve equations, inequalities, and systems of equations involving quadratic expressions and determine the values of the function it defines. Understand that polynomials form a system analogous to the integers, they choose and produce equivalent forms of an expression

Assessment Evidence (Stage 2)

Performance Task Description

Goal **Role** **Audience** **Situation** **Product/Performance** **Standards**	**Solving Linear Equations in Two Variables** http://map.mathshell.org/materials/download.php?fileid=694 This lesson unit is intended to help you assess how well students are able to formulate and solve problems using algebra and in particular, to identify and help students who have the following difficulties: • Solving a problem using two linear equations with two variables. • Interpreting the meaning of algebraic expressions. A-SSE: Interpret the structure of expressions. A-REI: Solve systems of equations. **Solving Linear Equations in Two Variables** http://map.mathshell.org/download.php?fileid=1730 This lesson unit is intended to help you assess how well students are able to formulate and solve problems using algebra and in particular, to identify and help students who have the following difficulties: • Solving a problem using two linear equations with two variables. • Interpreting the meaning of algebraic expressions. A-SSE: Interpret the structure of expressions. A-REI: Solve systems of equations. Sorting Equations and Identities http://map.mathshell.org/download.php?fileid=1720 This lesson unit is intended to help you assess how well students are able to: • Recognize the differences between equations and identities. • Substitute numbers into algebraic statements in order to test their validity in special cases. • Resist common errors when manipulating expressions such as $2(x - 3) = 2x - 3$; $(x + 3)^2 = x^2 + 3^2$. • Carry out correct algebraic manipulations.
Other Evidence	

Enage New York
Algebra I Module 4: Polynomial and Quadratic Expressions, Equations, and Function
llustrative Mathematics
http://www.illustrativemathematics.org/standard s/hs
http://www.wiki-teacher.com/ Math Resources – algebra

Learning Plan (Stage 3)

• **Where are your students headed? Where have they been? How will you make sure the students know where they are going?** • **How will you hook students at the beginning of the unit?** • **What events will help students experience and explore the big idea and questions in the unit? How will you equip them with needed skills and knowledge?** • **How will you cause students to reflect and rethink? How will you guide them in rehearsing, revising, and refining their work?** • **How will you help students to exhibit and self-evaluate their growing skills, knowledge, and understanding throughout the unit?** • **How will you tailor and otherwise personalize the learning plan to optimize the engagement and effectiveness of ALL students, without compromising the goals of the unit?** • **How will you organize and sequence the learning activities to optimize the engagement and achievement of ALL students?**	Have students create multiple ways to rewrite an expression that represents its equivalent form. Have them understand the notion of equivalent expression and the solution to an equation. Help them to understand that an equation in two variables can sometimes be viewed as defining a function, if one of the variables is designated as the input variable and the other as the output variable, and if there is just one output for each input. This is the case if the expression is of the form y = (expression in x) or if it can be put into the form by solving for y. • The use of algebraic tiles to establish a visual understanding of algebraic expression and the meaning of terms, factors, and coefficients can be effective. • The development and proper use of mathematical language (ie: Frayer Model, Word Wall, using real world context) could be used to introduce new terms. • Engage students in various techniques for solving quadratic equations and the relationship between those techniques (A-REI.4.a-b). Teach students to make use of the symmetric and transitive properties, and certain properties of equality with regards to operations (e.g. “equals added to equals is equal”) when solving equations. This approach would enable students to establish the idea of proof, while not explicitly named, is given a prominent role in the solving of equations, and the reasoning and justification process is not simply relegated to a future mathematics course. • Tile representations of quadratics illustrate that the process of completing the square has a geometric interpretation that explains the origin of the name. Encourage students to explore these representations in order to make sense out of the process of completing the square (MP.1, MP.5). Completing the square is an example of a theme that reoccurs throughout algebra: finding ways of transforming equations into certain standard forms that have the same solution

Unit 4	Function and Modeling	Grade Level	Level 3
Curriculum Area	Mathematics	Time Frame	3-4 weeks
Developed By	Munira Jamali		

Identify Desired Results (Stage 1)

Content Standards

Number and Quantity - The Real Number System

N.RN.3 Explain why the sum or product of two rational numbers is rational; that the sum of a rational number and an irrational number is irrational; and that the product of a nonzero rational number and an irrational number is irrational.

Functions - Interpreting Functions

F.IF.4 For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.

★ F.IF.5 Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. For example, if the function h(n) gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for he function.

★ F.IF.6 Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.★

F.IF.7 Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.★ a. Graph linear and quadratic functions and show intercepts, maxima, and minima.

b. Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value function.

F.IF.8 Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function. a. Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context. b. Use the properties of exponents to interpret expressions for exponential functions. For example, identify percent rate of change in functions such as $y = (1.02)^t$, $y = (0.97)^t$, , $y = (1.2)^{t/10}$ and classify them as representing exponential growth or decay.

F.IF.9 Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). For example, given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum.

Functions - Building Functions

F.BF.1 Write a function that describes a relationship between two quantities

.★ a. Determine an explicit expression,
a recursive process, or steps for calculation from a context.

b. Combine standard function types using arithmetic operations. For example, build a function that models the temperature of a cooling body by adding a constant function to a decaying exponential, and relate these functions to the model.

F.BF.3 Identify the effect on the graph of replacing f(x) by f(x) + k, k f(x), f(kx), and f(x + k) for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for the

Functions – Linear, Quadratic, and Exponential Model

F.LE.1 Distinguish between situations that can be modeled with linear functions and with exponential functions.

a. Prove that linear functions grow by equal differences over equal intervals; and that exponential functions grow by equal factors over equal intervals.

b. Recognize situations in which one quantity changes at a constant rate per unit interval relative to another.

c. Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another.

F.LE.2 Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).

F.LE.3 Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a

Understandings	Essential Questions	
Overarching Understanding	**Overarching**	**Topical**
Mathematical relationships can be presented graphically, in tables, or in verbal descriptions and the meaning of features in each representation can be interpreted in terms of the situation. Quadratic, linear or exponential function can be modeled, and the situation can be used in context to specify the domain and range as it relates to the understanding of real-world application of algebra concepts. The connection between the graph of the equation $y =$ (x) and the function itself can be made, and the coordinates of any point on the graph represent an input and output, expressed as $(x, (x))$. Translation between the tabular, graphical, and symbolic representations of a function can be explored between these representations and the situation's context. Key characteristics of functions can be identified, and function language and notation to analyze and compare functions can be used. The zeros and roots of a quadratic function can be solved by factoring or completing the square. Equivalent forms of linear, exponential and quadratic functions can be created to analyze and compare functions and features of functions (e.g. rates of change in specified intervals). The same function can be represented algebraically in different forms and the differences can be interpreted in terms of the graph or context. "The sum or product of two rational numbers is rational" can be explained, by arguing that the sum of two fractions with a integer numerator and denominator is also a fraction of the same type	How does each element of the domain correspond to exactly one element of the range? How would you relate and interpret features of relationships represented in a graph, table, and verbal descriptions? How can you represent the same function algebraically in different forms and interpret these differences in terms of the graph or context? What differences are there in the parameters of linear, exponential, and quadratic expressions? How would you model physical problems with linear, exponential and quadratic functions and what role would their parameters play in modeling? How can you find the zeros and roots of a quadratic function? How do the graphs of mathematical models and data help us better understand the world in which we live? How would you explain the product or sum of	How can you represent and describe functions? How can functions describe real-world situations, model predictions and solve problems? How do you interpret patterns in sequences, tables, and graphs using variables and functions? How do represent functions in different ways?
DIFFERENTIATION		

UDL/FRONT LOADING Prerequisites: Understanding and use the formal mathematical language of functions. Provide students an opportunity to compare two functions (quadratic and exponential), represented in different ways (table, graph, or situation). **ACCELERATION** Provide the students several opportunities to collect data to model different situations related to linear, quadratic, exponential functions, and trigonometric functions. Have students complete a project such as: The half-life of caffeine is 6 hours. In other words, after consuming some caffeine, half of that caffeine is still present in the body after 6 hours. The amount of caffeine in the body at the end of any given time interval is A=Pd-kt where P is the amount of caffeine present in the body at the beginning of the time interval, t is the length of the time interval, and k is the decay constant. For one day from the time you wake up to the time you go to bed, keep a record of the time and the amount consumed of any beverage that contains caffeine. Research how much caffeine is in each type of drink you consumed. Calculate the amount of caffeine in your body when you went to bed that night. Compare your results with your classmates. Use your calculations and the results of others to make a conjecture about the time of day you should consume your last caffeinated beverage if you want to have less than 20 milligrams in your body when you go to sleep. What time should you consume your last caffeinated beverage if you want to have no caffeine in your body when you go to sleep? (CORD Algebra 2: Learning in Context, 2008.) Which Function? – F.IF.8a This activity is a nice analysis that involves a real understanding of what the equation of a translated parabola looks like. **INTERVENTION** Have students evaluate different functions (linear, quadratics, and exponential) for a given variable. Then engage the students in identifying appropriate domain for the functions. Help students take the "function machine" that they learned in the earlier grades and turn it into a deeper understanding of relating the situation, table, and rule (formula) of functions. The goal here is to help students make the connections.	rational and irrational numbers?	
Knowledge Students will know…	**Skills** Students will be able to…	

To explore distinctions between rational and irrational numbers. To compare key characteristics of quadratic functions to those of linear and exponential functions. They select from among these functions to model phenomena. To anticipate the graph of a quadratic function by interpreting various forms of quadratic expressions. In particular, they identify the real solutions of a quadratic equation as the zeros of a related quadratic function. To learn that when quadratic equations do not have real solutions the number system must be extended so that solutions exist, analogous to the way in which extending the whole numbers to the negative numbers allows x+1 = 0 to have a solution. Formal work with complex numbers comes in Algebra II.	Use properties of rational and irrational numbers. Interpret functions that arise in applications in terms of a context. Focus on quadratic functions; compare with linear and exponential functions studied in Unit 2 Analyze functions using different representations To expand their experience with functions to include more specialized functions—absolute value, step, and those that are piecewise-defined. For F.IF.7b, compare and contrast absolute value, step and piecewise defined functions with linear, quadratic, and exponential functions. Highlight issues of domain, range, and usefulness when examining piecewise defined functions. Note that this unit, and in particular in F.IF.8b, extends the work begun in Unit 2 on exponential functions with integer exponents. For F.IF.9, focus on expanding the types of functions considered to include, linear, exponential, and quadratic. Extend work with quadratics to include the relationship between coefficients and roots, and that once roots are known, a quadratic equation can be factored. Build a function that models a relationship between two quantities. Focus on situations that exhibit a quadratic relationship. Build new functions from existing functions. For F.BF.3, focus on quadratic functions, and consider including absolute value functions. For F.BF.4a, focus on linear functions but consider simple situations where the domain of the function must be restricted in order for the inverse to exist, such as f(x) = x2, x>0 solve problems. Compare linear and exponential growth to quadratic growth. Interpret expressions for functions in terms of the situation they model
Assessment Evidence (Stage 2)	

Performance Task Description	
• **Goal** • **Role** • **Audience** • **Situation** • **Product/Performance** • **Standards**	Sorting Functions http://insidemathematics.org/common-core-math-tasks/high-school/HS-F-2008%20Sorting%20Functions.pdf Goal Your task is to match each graph with an equation, a table and a rule. Standards Understand the concept of a function and use function notation Functions and Everyday Situations http://map.mathshell.org/materials/download.php?fileid=1259 Goals Students are able to Articulate verbally the relationships between variables arising in everyday contexts. Translate between everyday situations and sketch graphs of relationships between variables. Interpret algebraic functions in terms of the contexts in which they arise. Reflect on the domains of everyday functions and in particular whether they should be discrete or continuous Standards F-IF: Interpret functions that arise in applications in terms of a context. Analyze functions using different representations. F-LE: Construct and compare linear, quadratic, and exponential models and solve problems. Interpret expressions for functions in terms of the situation they model. A-SSE: Interpret the structure of expressions
Other Evidence	

Influenza Epidemic – F.IF.4
http://www.illustrativemathematics.org/illustrations/637
• Warming and Cooling – F.IF.4:
http://www.illustrativemathematics.org/illustrations/639
• How is the weather – F.IF.4:
http://www.illustrativemathematics.org/illustrations/649
• Logistic Growth Model, Explicit Version – F.IF.4
http://www.illustrativemathematics.org/illustrations/804
• The Canoe Trip, Variation 1 – F.IF.4-5
http://www.illustrativemathematics.org/illustrations/386

• The High School Gym – F.IF.6b
http://www.illustrativemathematics.org/illustrations/577
• Temperature Change –F.IF.6
http://www.illustrativemathematics.org/illustrations/1500
• Which Function? - F.IF.8a
http://www.illustrativemathematics.org/illustrations/640
• Throwing Baseballs – F.IF.9 and F.IF.4
http://www.illustrativemathematics.org/illustrations/1279

Learning Plan (Stage 3)

• **Where are your students headed? Where have they been? How will you make sure the students know where they are going?** • **How will you hook students at the beginning of the unit?** • **What events will help students experience and explore the big idea and questions in the unit? How will you equip them with needed skills and knowledge?** • **How will you cause students to reflect and rethink? How will you guide them in rehearsing, revising, and refining their work?** • **How will you help students to exhibit and self-evaluate their growing skills, knowledge, and understanding throughout the unit?** • **How will you tailor and otherwise personalize the learning plan to optimize the engagement and effectiveness of ALL students, without compromising the goals of the unit?** • **How will you organize and sequence the learning activities to optimize the engagement and achievement of ALL students?**	Facilitate a discussion with students that would help them represent functions with graphs and identify key features in the graph. Create or used activity http://insidemathematics.org/common-core-math-tasks/high-school/HS-F-2008%20Sorting%20Functions.pdf where students would match different functions with their graphs, tables, and description. Engage students in graphing linear, exponential, and quadratic functions in order for them to develop fluency and the ability to graph them by hand. Help students to develop their idea of modeling physical problems with linear, exponential, and quadratic functions by looking at practical application of linear, quadratic, and exponential situations; such as stock market and investment, compound and simple interests, rocket trajectory, and speed of cars. Provide students the opportunity to compare linear, quadratic, and exponential functions, represented in different ways (table, graph, or situation) in writing using graphic organizers; such as T-chart or Venn diagram

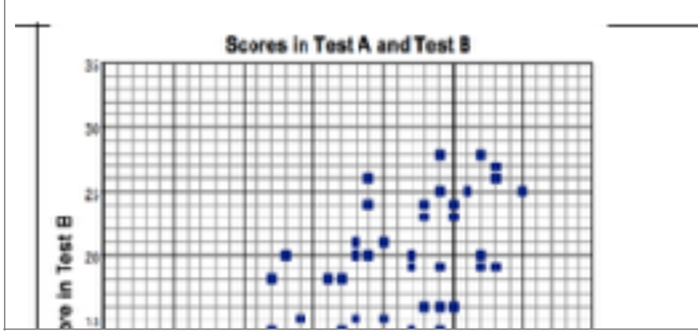
Unit 5	**Descriptive Statistics**	**8th grade**	Level 3
Mathematics		**Time Frame**	2-3 weeks
Munira Jamali			

Identify Desired Results (Stage 1)

Content Standards

Statistics and Probability - Interpreting Categorical and Quantitative Data
S.ID.1 Represent data with plots on the real number line (dot plots, histograms, and box plots)
. S.ID.2 Use statistics appropriate to the shape of the data distribution to compare center (median, mean) and spread (interquartile range, standard deviation) of two or more different data sets.
S.ID.3 Interpret differences in shape, center, and spread in the context of the data sets, accounting for possible effects of extreme data points (outliers)
S.ID.4 Use the mean and standard deviation of a data set to fit it to a normal distribution and to estimate population percentages
Statistics and Probability - Interpreting Categorical and Quantitative Data
S.ID.5 Summarize categorical data for two categories in two-way frequency tables. Interpret relative frequencies in the context of the data (including joint, marginal, and conditional relative frequencies). Recognize possible associations and trends in the data.
S.ID.6 Represent data on two quantitative variables on a scatter plot, and describe how the variables are related. a. Fit a function to the data; use functions fitted to data to solve problems in the context of the data. Use given functions or choose a function suggested by the context. Emphasize linear and exponential models. b. Informally assess the fit of a function by plotting and analyzing residuals. c. Fit a linear function for a scatter plot that suggests a linear association.
S.ID.7 Interpret the slope (rate of change) and the intercept (constant term) of a linear model in the context of the data.
S.ID.8 Compute (using technology) and interpret the correlation coefficient of a linear fit.
S.ID.9 Distinguish between correlation and causation.

Understandings	**Essential Questions**	
Overarching Understanding	**Overarching**	**Topical**
• A linear function can be used to model the relationship between two numerical variables. • The strength of a relationship and appropriateness of the model used can be determined by analyzing residuals. • A statistical relationship, such as correlation coefficient, is not necessarily the same as a cause and-effect relationship. • The correlation coefficient will be understood and the focus will be on the computation and interpretation of the correlation coefficient as a measure of how well the data fit the relationship. • A deeper look at bivariate data can be taken to describe categorical associations and how to fit models to quantitative data	How would you analyze bivariate data using your knowledge of proportions? How would you describe categorical variables? How would you use your knowledge of functions to fit models to quantitative data? How would you interpret the parameters of a linear model in the context of data that it represents? How can you compute correlation coefficients using technology and interpret the value of the coefficient? How do analysis of bivariate data and	How do you interpret data on a graph? How do you distinguish between a population and a sample? How do you design an experiment? How do you conduct a probability experiment? What is conditional probability? How do you determine if 2 events are mutually exclusive?
DIFFERENTIATION		

UDL/FRONT LOADING Use graphs of experiences that are familiar to students to increase accessibility and supports understanding and interpretation of proportional relationship. Students are expected to both sketch and interpret graphs including scatter plot. • Students create an equation with given information from a table, graph, or problem situation. • Engage students in interpreting slope and intercept using real world applications (e.g. bivariate data) **Acceleration** Students will explore how the residuals, the differences between the corresponding coordinates on the least squares line and the actual data values for each age, reveal additional information. Students should be able to sketch each distribution and answer questions about it just from knowledge of these three facts (shape, center, and spread). Have students design an experiment (project) where they would collect data from different sources, make a scatter plot of the data, draw a line of best fit modeling the data. From the plot, students would write the regression coefficient and the residual to explain the strength of the association. **Intervention** Have the students work in groups to generate data from the internet, such as the CST scores and other data. Have them construct a table based on the pattern and then graph the values and explain the relationship observed on the graph (association). Example: Certain students took two different tests (Test A and Test B). In the scatter diagram, each square represents one student and shows the scores that student got in the two tests 	bivariate data and knowledge of proportions intersect with each other?	exclusive?

Knowledge Students will know…	**Skills** Students will be able to…
To display numerical data and summarize it using measures of center and variability. By the end of middle school they were creating scatterplots and recognizing linear trends in data. To build upon that prior experience, providing students with more formal means of assessing how a model fits data. To use regression techniques to describe approximately linear relationships between quantities. To use graphical representations and knowledge of the context to make judgments about the appropriateness of linear models. With linear models, they look at residuals to analyze the goodness of fit.	Summarize, represent, and interpret data on a single count or measurement variable. In grades 6 – 8, students describe center and spread in a data distribution. Here they choose a summary statistic appropriate to the characteristics of the data distribution, such as the shape of the distribution or the existence of extreme data points Summarize, represent, and interpret data on two categorical and quantitative variables. Students take a more sophisticated look at using a linear function to model the relationship between two numerical variables. In addition to fitting a line to data, students assess how well the model fits by analyzing residuals. S.ID.6b should be focused on linear models, but may be used to preview quadratic functions in Unit 4 of this course Interpret linear models. Build on students' work with linear relationships in eighth grade and introduce the correlation coefficient. The focus here is on the computation and interpretation of the correlation coefficient as a measure of how well the data fit the relationship. The important distinction between a statistical relationship and a cause-and-effect relationship arises in S.ID.9

Assessment Evidence (Stage 2)

Performance Task Description

• **Goal** • **Role** • **Audience** • **Situation** • **Product/ Performance** • **Standards**	Representing Data with Frequency Graphs http://map.mathshell.org/materials/download.php?fileid=1230 Goals Students are able to Are able to use frequency graphs to identify a range of measures and make sense of this data in a real-world context. • Understand that a large number of data points allow a frequency graph to be approximated by a continuous distribution. Standard S-ID: Summarize, represent, and interpret data on a single count or measurement variable. Representing Data with Box Plots http://map.mathshell.org/download.php?fileid=1782 Students are able To interpret data using frequency graphs and box plots To identify and help students who have difficulty figuring out the data points and spread of data from frequency graphs and box plots.

Other Evidence

Interpreting Statistics: A Case of Muddying the Waters – S.ID 7-9
http://map.mathshell.org/materials/download.php?fileid=686
ILLUSTRIVE MATHEMATICS
• Speed Trap – S.ID.1, 2, 3:
http://www.illustrativemathematics.org/illustrations/1027
Coffee and Crime – S.ID.6-9:
http://www.illustrativemathematics.org/illustrations/1307
• Olympic Men's 100-meter dash – S.ID.6a, 7:
http://www.illustrativemathematics.org/illustrations/1554
• Used Subaru Foresters I – S.ID.6a:
http://www.illustrativemathematics.org/illustrations/941
• Texting and Grades II – S.ID.7
https://www.illustrativemathematics.org/content-standards/tasks/1028

Learning Plan (Stage 3)

• **Where are your students headed? Where have they been? How will you make sure the students know where they are going?** • **How will you hook students at the beginning of the unit?** • **What events will help students experience and explore the big idea and questions in the unit? How will you equip them with needed skills and knowledge?** • **How will you cause students to reflect and rethink? How will you guide them in rehearsing, revising, and refining their work?** • **How will you help students to exhibit and self-evaluate their growing skills, knowledge, and understanding throughout the unit?** • **How will you tailor and otherwise personalize the learning plan to optimize the engagement and effectiveness of ALL students, without compromising the goals of the unit?** • **How will you organize and sequence the learning activities to optimize the engagement and achievement of ALL students?**	Use graphs such as the one below to show two ways of comparing height data for males and females in the 20-29 age group. Both involve plotting the data or data summaries (box plots or histograms) on the same scale, resulting in what are called parallel (or side-by-side) box plots and parallel histograms (S-ID. 1). The parallel histograms show the distributions of heights to be mound shaped and fairly symmetrical (approximately normal) in shape. The data can be described using the mean and standard deviation. Have students sketch each distribution and answer questions about it just from knowledge of these three facts (shape, center, and spread). They also observe that the two measures of center, median and mean, tend to be close to each other for symmetric distributions Have students learn how to take a careful look at scatter plots, as sometimes the "obvious" pattern does not tell the whole story, and can even be misleading. The graphs show the median heights of growing boys through the ages 2 to 14. The line (least squares regression line) with slope 2.47 inches per year of growth looks to be a perfect fit (S-ID.6c). But, the residuals, the differences between the corresponding coordinates on the least squares line and the actual data values for each age, reveal additional information (such as a teacher think-aloud). Sample questions to facilitate student discussion and understanding: What does this scatter plot/histogram show? How do you know? Pick any point on the histogram and explain what it means. What does it mean in relation to the other plots on the histogram? Does anyone have another way to explain it?