Geometry

	Key Standards Covered	Possible Resources

Quarter 1 September 6- November 2	• **G.CO.1** Know precise definitions of angle, circle, perpendicular lines, parallel lines, and line segment, based on the undefined notions of point, line, distance along a line, and distance around a circular arc. • **G.CO.2** Represent transformations in the plane using e.g. transparencies and geometry software; describe transformations as functions that take points in the plane as inputs and give other points as outputs. Compare transformations that preserve distance and angle to those that do not (e.g. translation versus horizontal stretch.) • **G.CO.3** Given a rectangle, parallelogram, trapezoid, or regular polygon, describe the rotations and reflections that carry it onto itself. • **G.CO.4** Develop definitions of rotations, reflections, and translations in terms of angles, circles perpendicular lines, parallel lines, and line segments. • **G.CO.5** Given a geometric figure and a rotation, reflection or translation, draw the transformed figure using e.g. graph paper, tracing paper, or geometry software. Specify a sequence of transformations that will carry a given figure onto another • **G.CO.6** Use geometric descriptions of rigid motions to transform figures and to predict the effect of a given rigid motion on a given figure; given two figures, use the definition of congruence in terms of rigid motions to decide if they are congruent. • **G.CO.7** Use definition of congruence in terms of rigid motions to show that two triangles are congruent if and only if corresponding pairs of sides and corresponding pairs of angles are congruent. • **G.CO.8** Explain how the criteria for triangle congruence (ASA, SAS, and SSS) follow the	• Aaron's Design http://map.mathshell.org/materials/download.php?fileid=1054 • Possible Triangle Constructions http://map.mathshell.org/materials/lessons.php?taskid=581&subpage=concept • Transforming 2D Figures http://map.mathshell.org/materials/download.php?fileid=1538 • Illustrative Mathematics: "Points equidistant from two points in the plane" http://www.illustrativemathematics.org/illustrations/967 • Illustrative Mathematics: "Tangent Lines and the Radius of a Circle" http://www.illustrativemathematics.org/illustrations/963 • PARCC – http://www.parcconline.org/samples/ma thematics/high-school-mathematics http://www.parcconline.org/sites/parcc/files/PARCC_SampleItems_Mathematics_HSGeoMathIIIGeometricConnection _081913_Final_0.pd

	Key Standards Covered	Possible Resources

Quarter 2 November 12- January 28	● **G..GPE.6 :** Find the point on a directed line segment between two given points that partitions the segment in a given ratio. ● **G.SRT1a:** Verify experimentally the properties of dilations given by a center and a scale factor: a dilation takes a line not passing through the center of the dilation to a parallel line, and leaves a line passing through the center unchanged ● **G.SRT1b:** The dilation of a line segment is longer or shorter in the ratio given by the scale factor. ● **G.SRT.2** Given two figures, use the definition of similarity in terms of similarity transformations to decide if they are similar; explain using similarity transformations the meaning of similarity for triangles as the equality of all corresponding pairs of angles and the proportionality of all corresponding pairs of sides. ● **G.CO.10:** *Prove theorems about triangles. Theorems include: measures of interior angles of a triangle sum to 180°; base angles of isosceles triangles are congruent; the segment joining midpoints of two sides of a triangle is parallel to the third side and half the length; the medians of a triangle meet at a point.* ● **G.SRT.3**Use the properties of similarity transformations to establish the Angle-Angle (AA) criterion for two triangles to be similar	● Illustrative Mathematics: "Similar Triangles" http://www.illustrativemathematics.org/illustrations/1422 ● "Solving Geometry Problems: Floodlights" – Mathematics Assessment Project ● This lesson unit is intended to help you assess how well students are able to identify and use geometrical knowledge to solve a problem. In particular, this unit aims to identify and help students who have difficulty in: •Making a mathematical model of a geometrical situation. •Drawing diagrams to help with solving a problem. •Identifying similar triangles and using their properties to solve problems. •Tracking and reviewing strategic decisions when problem-solving. ● http://map.mathshell.org/materials/lessons.php?taskid=429#task429 ● PARCC – http://www.parcconline.org/samples/ma thematics/high-school-mathematics http://www.parcconline.org/sites/parcc/files/PARCC_SampleItems_Mathematics_HSGeoMathIIIGeometricConnecti

	Key Standards Covered	Possible Resources

<table>
<tr>
<td>Quarter 3
February 4-
April 5</td>
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G.GPE.6 : Find the point on a directed line segment between two given points that partitions the segment in a given ratio
G.GPE.7: Use coordinates to compute perimeters of polygons and areas of triangles and rectangles, e.g., using the distance formula.*
G.SRT.4: Prove theorems about triangles. Theorems include: a line parallel to one side of a triangle divides the other two proportionally, and conversely; the Pythagorean Theorem proved using triangle similarity.
G.SRT.5: Use congruence and similarity criteria for triangles to solve problems and to prove relationships in geometric figures.
G.SRT.9 : Derive the formula $A = 1/2\ ab \sin(C)$ for the area of a triangle by drawing an auxiliary line from a vertex perpendicular to the opposite side.
G.SRT.8 : Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems.
G.SRT.10: Prove the Laws of Sines and Cosines and use them to solve problems.
G.SRT.11: Understand and apply the Law of Sines and the Law of Cosines to find unknown measurements in right and non-right triangles (e.g., surveying problems, resultant forces).

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<td>

“Solving Geometry Problems: Floodlights” – Mathematics Assessment Project
This lesson unit is intended to help you assess how well students are able to identify and use geometrical knowledge to solve a problem. In particular, this unit aims to identify and help students who have difficulty in:
•Making a mathematical model of a geometrical situation.
Drawing diagrams to help with solving a problem.
Identifying similar triangles and using their properties to solve problems.
Tracking and reviewing strategic decisions when problem-solving.
http://map.mathshell.org/materials/lessons.php?taskid=429#task429
“Proofs of the Pythagorean Theorem” – Mathematics Assessment Project
This lesson unit is intended to help you assess how well students are able to produce and evaluate geometrical proofs.
In particular, this unit is intended to help you identify and assist students who have difficulties in:
•Interpreting diagrams.
•Identifying mathematical knowledge

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</table>

	Key Standards Covered	Possible Resources

Quarter 4 April 8- June 17	● **G.C.2**. Identify and describe relationships among inscribed angles, radii, and chords. Include the relationship between central, inscribed, and circumscribed angles; inscribed angles on a diameter are right angles; the radius of a circle is perpendicular to the tangent where the radius intersects the circle. ● **G.C.5**Derive using similarity the fact that the length of the arc intercepted by an angle is proportional to the radius, and define the radian measure of the angle as the constant of proportionality; derive the formula for the area of a sector. Convert between degrees and radians. CA ● **G.MG.1** Use geometric shapes, their measures, and their properties to describe objects (e.g., modeling a tree trunk or a human torso as a cylinder).* ● **G.GPE.4**Use coordinates to prove simple geometric theorems algebraically. For example, prove or disprove that a figure defined by four given points in the coordinate plane is a rectangle; prove or disprove that the point (1, 3) lies on the circle centered at the origin and containing the point (0, 2) ● **G.GPE.5**. Prove the slope criteria for parallel and perpendicular lines and use them to solve geometric problems (e.g., find the equation of a line parallel or perpendicular to a given line that passes through a given point) ● **G.GC.3**Construct the inscribed and circumscribed circles of a triangle, and prove properties of angles for a quadrilateral inscribed in a circle ● **G.GPE.7**Use coordinates to compute perimeters of polygons and areas of triangles and rectangles, e.g., using the distance formula ● **G.GPE.1**Derive the equation of a circle of given	● Illustrative Mathematics: “Right triangles inscribed in circles I” ● http://www.illustrativemathematics.org/illustrations/1091 ● Illustrative Mathematics: “Right triangles inscribed in circles II” ● http://www.illustrativemathematics.org/illustrations/1093 ● llustrative Mathematics: “Tangent Lines and the Radius of a Circle” ● http://www.illustrativemathematics.org/illustrations/963 ● Illustrative Mathematics: “Neglecting the Curvature of the Earth” ● http://www.illustrativemathematics.org/illustrations/1345 ● “Equations of Circles 1” – Mathematics Assessment Project ● This lesson unit is intended to help you assess how well students are able to: •Use the Pythagorean theorem to derive the equation of a circle. •Translate between the geometric features of circles and their equations. ● http://map.mathshell.org/materials/lessons.php?taskid=406#task406

Title of Unit	Congruence , Proof and Construction	Grade Level	10 th grade Geometry
Curriculum Area	Math	Time Frame	6-7 weeks
Developed By	Munira Jamali		

Identify Desired Results (Stage 1)

Content Standards

G.CO.1 Know precise definitions of angle, circle, perpendicular lines, parallel lines, and line segment, based on the undefined notions of point, line, distance along a line, and distance around a circular arc.
G.CO.2 Represent transformations in the plane using e.g. transparencies and geometry software; describe transformations as functions that take points in the plane as inputs and give other points as outputs. Compare transformations that preserve distance and angle to those that do not (e.g. translation versus horizontal stretch.)
G.CO.3 Given a rectangle, parallelogram, trapezoid, or regular polygon, describe the rotations and reflections that carry it onto itself.
G.CO.4 Develop definitions of rotations, reflections, and translations in terms of angles, circles perpendicular lines, parallel lines, and line segments.
G.CO.5 Given a geometric figure and a rotation, reflection or translation, draw the transformed figure using e.g. graph paper, tracing paper, or geometry software. Specify a sequence of transformations that will carry a given figure onto another
G.CO.6 Use geometric descriptions of rigid motions to transform figures and to predict the effect of a given rigid motion on a given figure; given two figures, use the definition of congruence in terms of rigid motions to decide if they are congruent.
G.CO.7 Use definition of congruence in terms of rigid motions to show that two triangles are congruent if and only if corresponding pairs of sides and corresponding pairs of angles are congruent.
G.CO.8 Explain how the criteria for triangle congruence (ASA, SAS, and SSS) follow the definition of congruence in terms of rigid motion Explain how the criteria for triangle congruence (ASA, SAS, and SSS) follow the definition of congruence in terms of rigid motion
G.CO.9. Prove theorems about lines and angles. Theorems include: vertical angles are congruent; when a transversal crosses parallel lines, alternate interior angles are congruent and corresponding angles are congruent; points on a perpendicular bisector of a line segment are exactly those equidistant from the segment's endpoints
G.CO.10. Prove theorems about triangles. Theorems include: measures of interior angles of a triangle sum to 180°; base angles of isosceles triangles are congruent; the segment joining midpoints of two sides of a triangle is parallel to the third side and half the length; the medians of a triangle meet at a point.
G.CO.11 Prove theorems about parallelograms. Theorems include: opposite sides are congruent, opposite angles are congruent; the diagonals of a parallelogram bisect each other, and conversely, rectangles are parallelograms with congruent diagonals.
G.CO.12 Make formal geometric constructions with a variety of tools and methods (compass and straightedge, string, reflective devices, paper folding, dynamic geometric software etc. Copying a segment, copying an angle; bisecting a segment; bisecting an angle; constructing perpendicular lines including the perpendicular bisector of a line segment; and constructing a line parallel to a give line through a point not on the line.
G.CO.13 Construct an equilateral triangle, a square, a regular hexagon inscribed in a circle.

<table>
<tr><th>Understandings</th><th colspan="2">Essential Questions</th></tr>
<tr><th>Overarching Understanding</th><th>Overarching</th><th>Topical</th></tr>
<tr><td>The fundamental tools of classic construction are the compass and the straightedge but, there are many other tools useful for constructions including; string, reflective devices, protractor, and geometric software.
Geometric construction is a visual representation of geometric principals and develops a deeper understanding of the spatial relationships between pairs of figures and their elements.
Transformations include a variety of motions that take a set of points in the plane as input and gives us other points as output.
There are rigid transformations that preserve distance and angles and non-rigid transformations that do not.
The properties of transformations that are rigid motion can be used to identify and prove congruence of figures in a plane.
Constructing a viable argument using the precise vocabulary of transformations and congruence to prove geometric theorems in a variety of formats is important to Geometry proof</td><td rowspan="2">How do geometric constructions relate to geometric to geometric reasoning and proof?
• What are the justifications that can be used to guide geometric constructions?
• What are the criteria that can be used by a geometry student to select the most appropriate tools and software for geometric constructions?
• What are the similarities and differences among the various transformations and how can they be grouped as either rigid or non-rigid?</td><td rowspan="2">How would you construct angle bisector from a given angle?

How would you construct regular hexagon?

How would you perpendicular bisector of a line?</td></tr>
<tr><th>Related Misconceptions</th></tr>
</table>

For Standards : G.CO.1-5 The terms "mapping" and "under" are used in special ways when studying transformations. A translation is a type of transformation that moves all the points in the object in a straight line in the same direction. Students should know that not every transformation is a translation. Students sometimes confuse the terms "transformation" and "translation." **G.CO.6-8** Some students may believe: That combinations such as SSA or AAA are also a congruence criterion for triangles. Provide counterexamples for this misconception. That all transformations, including dilations, are rigid motions. Provide counterexamples for this misconception. That any two figures that have the same area represent a rigid transformation. Students should recognize that the areas remain the same, but preservation of side and angle lengths determine that the transformation is rigid. That corresponding vertices do not have to be listed in order; however, it is useful to stress the importance of listing corresponding vertices in the same order so that corresponding sides and angles can be easily identified and that included sides or angles are apparent. **G.CO.9-11** Research over the last four decades suggests that student misconceptions about proof abound: • even after proving a generalization, students believe that exceptions to the generalization might exist; • one counterexample is not sufficient; • the converse of a statement is true (parallel lines do not intersect, lines that do not intersect are parallel); and • a conjecture is true because it worked in all examples that were explored. Each of these misconceptions needs to be addressed, both by the ways in which formal proof is taught in geometry and how ideas about "justification" are developed throughout a student's mathematical education **G.CO.12-13** Some students may believe that a construction is the same as a sketch or drawing. Emphasize the need for precision and accuracy when doing constructions. Stress the idea that a compass and straightedge	• How can the properties of rigid motion be used to prove that two triangles are congruent (ASA, SAS, SSS)? • What are the various pathways to create a valid proof for theorems about lines, angles, triangles congruence and parallelograms?	

Knowledge Students will know…	**Skills** Students will be able to…
1. To make a variety of formal geometric constructions using a variety of tools 2. To develop precise definitions of geometric figures based on the undefined notions of point, line, distance along a line and distance around a circular arc. Experiment with transformations in the plane. 3. To use rigid motion to map corresponding parts of congruent triangle onto each other. Explain triangle congruence in terms of rigid motions. 4. Prove theorems about lines and angles, triangles; and parallelograms.	1. Make geometric construction 2. Experiment with transformations in the plane 3. Understand congruence in terms of rigid motions 4. Prove geometric theorems

Assessment Evidence (Stage 2)

Performance Task Description

• **Goal** • **Role** • **Audience** • **Situation** • **Product/Performance** • **Standards**	Aaron's Design http://map.mathshell.org/materials/download.php?fileid=1054 Possible Triangle Constructions http://map.mathshell.org/materials/lessons.php?taskid=581&subpage=concept Transforming 2D Figures http://map.mathshell.org/materials/download.php?fileid=1538 Illustrative Mathematics: "Points equidistant from two points in the plane" http://www.illustrativemathematics.org/illustrations/967 Illustrative Mathematics: "Tangent Lines and the Radius of a Circle" http://www.illustrativemathematics.org/illustrations/963

Other Evidence

PARCC – http://www.parcconline.org/samples/ma thematics/high-school-mathematics http://www.parcconline.org/sites/parcc/files/PARCC_SampleItems_Mathematic s_HSGeoMathIIIGeometricConnection _081913_Final_0.pd
Learning Plan (Stage 3)

- **Where are your students headed? Where have they been? How will you make sure the students know where they are going?**
- **How will you hook students at the beginning of the unit?**
- **What events will help students experience and explore the big idea and questions in the unit? How will you equip them with needed skills and knowledge?**
- **How will you cause students to reflect and rethink? How will you guide them in rehearsing, revising, and refining their work?**
- **How will you help students to exhibit and self-evaluate their growing skills, knowledge, and understanding throughout the unit?**
- **How will you tailor and otherwise personalize the learning plan to optimize the engagement and effectiveness of ALL students, without compromising the goals of the unit?**
- **How will you organize and sequence the learning activities to optimize the engagement and achievement of ALL students?**

For Standard : G.CO.2

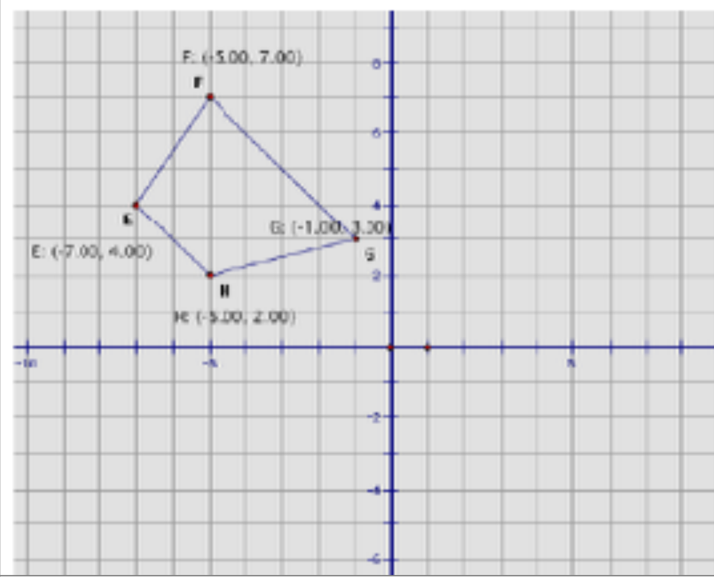
In middle school students have worked with translations, reflections, and rotations and informally with dilations. Point out the basis of rigid motions in geometric concepts, e.g., translations move points a specified distance along a line parallel to a specified line; rotations move objects along a circular arc with a specified center through a specified angle. Use various technologies such as transparencies, geometry software, interactive whiteboards, and digital visual presenters to represent and compare rigid and size transformations of figures in a coordinate plane. Comparing transformations that preserve distance and angle to those that do not. Describe and compare function transformations on a set of points as inputs to produce another set of points as outputs, to include translations and horizontal and vertical stretching. Students may use geometry software and/or manipulatives to model and compare transformations. Examples:

• Draw transformations of reflections, rotations, translations, and combinations of these using graph paper, transparencies and/or geometry software.

• Determine the coordinates for the image (output) of a figure when a transformation rule is applied to the preimage (input).

• Distinguish between transformations that are rigid (preserve distance and angle measure-reflections, rotations, translations, or combinations of these) and those that are not (dilations or rigid motions followed by dilations)

• The figure below is reflected across the y-axis and then shifted up by 4 units. Draw the transformed figure and label the new coordinates. What function can be used to describe these transformations in the coordinate plane?



From: Wiggins, Grant and J. Mc Tighe. (1998). Understanding by Design, Association for Supervision and Curriculum Development
ISBN # 0-87120-313-8 (ppk)

Title of Unit	Similarity and Proof	**Grade Level**	10th grade Geometry
Curriculum Area	Mathematics	**Time Frame**	6-7 weeks
Developed By	Munira Jamali		

Identify Desired Results (Stage 1)

Content Standards

G..GPE.6 : Find the point on a directed line segment between two given points that partitions the segment in a given ratio.
G.SRT1a: Verify experimentally the properties of dilations given by a center and a scale factor: a dilation takes a line not passing through the center of the dilation to a parallel line, and leaves a line passing through the center unchanged
G.SRT1b: The dilation of a line segment is longer or shorter in the ratio given by the scale factor.
G.SRT.2 Given two figures, use the definition of similarity in terms of similarity transformations to decide if they are similar; explain using similarity transformations the meaning of similarity for triangles as the equality of all corresponding pairs of angles and the proportionality of all corresponding pairs of sides.
G.CO.10: *Prove theorems about triangles. Theorems include: measures of interior angles of a triangle sum to 180°; base angles of isosceles triangles are congruent; the segment joining midpoints of two sides of a triangle is parallel to the third side and half the length; the medians of a triangle meet at a point*.
G.SRT.3Use the properties of similarity transformations to establish the Angle-Angle (AA) criterion for two triangles to be similar

Understandings	**Essential Questions**	
Overarching Understanding	**Overarching**	**Topical**

Sequence of similarity transformation of two objects that maps one exactly onto the other is defined. • Similarity of two objects using their given ratio by a scale factor is proved; such as: using the dilation of a line segment in ratio given by the scale factor. • Similar triangles have corresponding pairs of angles and proportional pairs of sides (AA, SAS, SSS). • Prove Theorems about triangles; such as "a line parallel to one side of a triangle divides the other two proportionately and conversely." • Triangle similarity is used to prove the Pythagorean Theorem. • Congruence and similarity criteria for triangles are used to solve problems and prove relationships of geometric figures.	• What is the difference between similarity and congruence? • How can you show that it is not possible to prove similarity by showing three angles in proportion to one another? • How do you construct a viable argument for congruency and/or similarity of two triangles? • How do you construct a viable argument for the similarity of geometric figures? • Are all congruent triangles similar and is the converse true also?	How are two geometric figures similar? What is proportionality rule? What is Pythagorean theorem ?
Related Misconceptions		

G.SRT.1-3 Some students often do not recognize that congruence is a special case of similarity. Similarity with a scale factor equal to 1 becomes a congruency. Students may not realize that similarities preserve shape, but not size. Angle measures stay the same, but side lengths change by a constant scale factor. Students may incorrectly apply the scale factor. For example students will multiply instead of divide with a scale factor that reduces a figure or divide instead of multiply when enlarging a figure. Some students often do not list the vertices of similar triangles in order. However, the order in which vertices are listed is preferred and especially important for similar triangles so that proportional sides can be correctly identified.		

Knowledge Students will know…	**Skills** Students will be able to…
• To investigate triangles and decide when they are similar. • To develop more precise mathematical definition of similarity; the new definition taken for two objects being similar is that there is a sequence of similarity transformations that maps one exactly onto the other. • To explore the consequences of two triangles being similar: that they have congruent angles and that their side lengths are in the same proportion. Students prove the Pythagorean Theorem using triangle similarity. • Prove geometric theorems	• Understand similarity in terms of similarity transformations. • Prove theorems involving similarity • Prove theorems about lines and angles, triangles; and parallelograms.

Assessment Evidence (Stage 2)

Performance Task Description

• **Goal** • **Role** • **Audience** • **Situation** • **Product/Performance** • **Standards**	Illustrative Mathematics: "Similar Triangles" http://www.illustrativemathematics.org/illustrations/1422 "Solving Geometry Problems: Floodlights" – Mathematics Assessment Project This lesson unit is intended to help you assess how well students are able to identify and use geometrical knowledge to solve a problem. In particular, this unit aims to identify and help students who have difficulty in: •Making a mathematical model of a geometrical situation. •Drawing diagrams to help with solving a problem. •Identifying similar triangles and using their properties to solve problems. •Tracking and reviewing strategic decisions when problem-solving. http://map.mathshell.org/materials/lessons.php?taskid=429#task429

Other Evidence

PARCC –
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Learning Plan (Stage 3)

- **Where are your students headed? Where have they been? How will you make sure the students know where they are going?**
- **How will you hook students at the beginning of the unit?**
- **What events will help students experience and explore the big idea and questions in the unit? How will you equip them with needed skills and knowledge?**
- **How will you cause students to reflect and rethink? How will you guide them in rehearsing, revising, and refining their work?**
- **How will you help students to exhibit and self-evaluate their growing skills, knowledge, and understanding throughout the unit?**
- **How will you tailor and otherwise personalize the learning plan to optimize the engagement and effectiveness of ALL students, without compromising the goals of the unit?**
- **How will you organize and sequence the learning activities to optimize the engagement and achievement of ALL students?**

Instructional Strategies: G.SRT.1-3

Allow adequate time and hands-on activities for students to explore dilations visually and physically.

Use graph paper and rulers or dynamic geometry software to obtain images of a given figure under dilations having specified centers and scale factors. Carefully observe the images of lines passing through the center of dilation and those not passing through the center, respectively. A line segment passing through the center of dilation will simply be shortened or elongated but will lie on the same line, while the dilation of a line segment that does not pass through the center will be parallel to the original segment (this is intended as a clarification of Standard 1a).

Illustrate two-dimensional dilations using scale drawings and photocopies.

Measure the corresponding angles and sides of the original figure and its image to verify that the corresponding angles are congruent and the corresponding sides are proportional (i.e. stretched or shrunk by the same scale factor).

Investigate the SAS and SSS criteria for similar triangles.

Use graph paper and rulers or dynamic geometry software to obtain the image of a given figure under a combination of a dilation followed by a sequence of rigid motions (or rigid motions followed by dilation).

Work backwards – given two similar figures that are related by dilation, determine the center of dilation and scale factor. Given two similar figures that are related by a dilation followed by a sequence of rigid motions, determine the parameters of the dilation and rigid motions that will map one onto the other.

Using the theorem that the angle sum of a triangle is 180°, verify that the AA criterion is equivalent to the AAA criterion. Given two triangles for which AA holds, use rigid motions to map a vertex of one triangle onto the corresponding vertex of the other in such a way that their corresponding sides are in line. Then show that dilation will complete the mapping of one triangle onto the other.

Students may be interested in scale models or experiences with blueprints and scale drawings (perhaps in a work related situation) to illustrate similarity.

Explanations and Examples: G.SRT.1

Students should understand that a dilation is a transformation that moves each point along the ray through the point emanating from a fixed center, and multiplies distances from the center by a common scale factor.

Perform a dilation with a given center and scale factor on a figure in the coordinate plane. Verify that when a side passes through the center of dilation, the side and its image lie on the same line. Verify that corresponding sides of the preimage and images are parallel. Verify that a side length of the image is equal to the scale factor multiplied by the corresponding side length of the preimage.

Title of Unit	Trigonometry	**Grade Level**	10th grade geometry
Curriculum Area	Math	**Time Frame**	5-6 weeks
Developed By	Munira Jamali		
Identify Desired Results (Stage 1)			
Content Standards			

G.GPE.6 : Find the point on a directed line segment between two given points that partitions the segment in a given ratio
G.GPE.7: Use coordinates to compute perimeters of polygons and areas of triangles and rectangles, e.g., using the distance formula.*
G.SRT.4: Prove theorems about triangles. *Theorems include: a line parallel to one side of a triangle divides the other two proportionally, and conversely; the Pythagorean Theorem proved using triangle similarity.*
G.SRT.5: Use congruence and similarity criteria for triangles to solve problems and to prove relationships in geometric figures.
G.SRT.9 : Derive the formula $A = 1/2\ ab \sin(C)$ for the area of a triangle by drawing an auxiliary line from a vertex perpendicular to the opposite side.
G.SRT.8 : Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems.

G.SRT.10: Prove the Laws of Sines and Cosines and use them to solve problems.

G.SRT.11: Understand and apply the Law of Sines and the Law of Cosines to find unknown measurements in right and non-right triangles (e.g., surveying problems, resultant forces).

Understandings	Essential Questions	
Overarching Understanding	**Overarching**	**Topical**
•Express a geometric relationship algebraically (e.g. the Pythagorean Theorem) to new situations such as deriving equation of a circle using the distance formula or deriving the equation of a parabola in terms of focus and directrix. •Right triangle and triangle similarity can be applied to geometric and algebraic theorems to find coordinates of a point on a line given proportion of segments on the line. • Justify algebraically the relationships between slopes of parallel and perpendicular lines as they can be established through proof. • The algebraic representation of a geometric problem can be used to prove theorems in a coordinate plane •Understand trigonometric ratios as the relationships between sides and angles in right triangles. • Understand the concept of complementary angles through sine and cosine. • Trigonometric ratios can be derived for special right triangles (30-60-90 and 45-45-90). • Real world problems can be solved using right triangles, trigonometric ratios and the Pythagorean theorem.	• Given coordinate plane information, can we prove (or disprove) geometric relationships (e.g. given the vertices, disprove the assertion that ABCD is a rhombus; or that a given point lies on a circle)? • What is always true about the slopes of perpendicular (or, parallel) lines, and how can a proof be written to exemplify this?	What is right angle similarity? How is law of sines and cosines used in real life application? What is trigonometric real life application?
Related Misconceptions		

G.SRT.6-8 Some students believe that right triangles must be oriented a particular way. Some students do not realize that opposite and adjacent sides need to be identified with reference to a particular acute angle in a right triangle. Some students believe that the trigonometric ratios defined in this cluster apply to all triangles, but they are only defined for acute angles in right triangles.		

Knowledge Students will know…	**Skills** Students will be able to…
1. To investigate triangles and decide when they are similar; with this newfound knowledge and their prior understanding of proportional relationships, they define trigonometric ratios and solve problems using right triangles. 2. To explore right triangle trigonometry, and circles and parabolas. Throughout the course, Mathematical Practice 3, "Construct viable arguments and critique the reasoning of others," plays a predominant role. 3. To advance their knowledge of right triangle trigonometry by applying trigonometric ratios in non-right triangles. 4. Prove the Laws of Sines and Cosines and use them to solve problems	Use coordinates to prove simple geometric theorems algebraically. Define trigonometric ratios and solve problems involving right triangles.

Assessment Evidence (Stage 2)

Performance Task Description

• **Goal** • **Role** • **Audience** • **Situation** • **Product/Performance** • **Standards**	"Solving Geometry Problems: Floodlights" – Mathematics Assessment Project This lesson unit is intended to help you assess how well students are able to identify and use geometrical knowledge to solve a problem. In particular, this unit aims to identify and help students who have difficulty in: •Making a mathematical model of a geometrical situation. •Drawing diagrams to help with solving a problem. •Identifying similar triangles and using their properties to solve problems. •Tracking and reviewing strategic decisions when problem-solving. http://map.mathshell.org/materials/lessons.php?taskid=429#task429 G.SRT.4 "Proofs of the Pythagorean Theorem" – Mathematics Assessment Project This lesson unit is intended to help you assess how well students are able to produce and evaluate geometrical proofs. In particular, this unit is intended to help you identify and assist students who have difficulties in: •Interpreting diagrams. •Identifying mathematical knowledge relevant to an argument. •Linking visual and algebraic representations. •Producing and evaluating mathematical arguments. http://map.mathshell.org/materials/lessons.php?taskid=419#task419 Illustrative Mathematics: "Joining two midpoints of sides of a triangle" http://www.illustrativemathematics.org/illustrations/1095 Illustrative Mathematics: "Pythagorean Theorem" http://www.illustrativemathematics.org/illustrations/1568 "Analyzing Congruence Proofs" – Mathematics Assessment Project This lesson unit is intended to help you assess how well students are able to: •Work with concepts of congruency and similarity, including identifying corresponding sides and corresponding angles within and between triangles. •Identify and understand the significance of a counter-example. •Prove, and evaluate proofs in a geometric context.. http://map.mathshell.org/materials/lessons.php?taskid=452#task452 Illustrative Mathematics: "Bank Shot" http://www.illustrativemathematics.org/illustrations/651 Illustrative Mathematics: "Extensions, Bisections and Dissections in a Rectangle" http://www.illustrativemathematics.org/illustrations/1009 Illustrative Mathematics: "Folding a square into thirds" http://www.illustrativemathematics.org/illustrations/1572 Illustrative Mathematics: "Tangent Line to Two Circles" http://www.illustrativemathematics.org/illustrations/916 Illustrative Mathematics: "Congruence of parallelograms" http://www.illustrativemathematics.org/illustrations/1517 **Tools/Resources:** **G.SRT.6-8** "Geometry Problems: Circles and Triangles" – Mathematics Assessment Project http://map.mathshell.org/materials/lessons.php?taskid=222#task222

Other Evidence
PARCC – http://www.parcconline.org/samples/ma thematics/high-school-mathematics http://www.parcconline.org/sites/parcc/files/PARCC_SampleItems_Mathematic s_HSGeoMathIIIGeometricConnection _081913_Final_0.pd

Learning Plan (Stage 3)

• **Where are your students headed? Where have they been? How will you make sure the students know where they are going?** • **How will you hook students at the beginning of the unit?** • **What events will help students experience and explore the big idea and questions in the unit? How will you equip them with needed skills and knowledge?** • **How will you cause students to reflect and rethink? How will you guide them in rehearsing, revising, and refining their work?** • **How will you help students to exhibit and self-evaluate their growing skills, knowledge, and understanding throughout the unit?** • **How will you tailor and otherwise personalize the learning plan to optimize the engagement and effectiveness of ALL students, without compromising the goals of the unit?** • **How will you organize and sequence the learning activities to optimize the engagement and achievement of ALL students?**	**Instructional Strategies: G.SRT.4-5** Review triangle congruence criteria and similarity criteria, if it has already been established. Review the angle sum theorem for triangles, the alternate interior angle theorem and its converse, and properties of parallelograms. Visualize it using dynamic geometry software. Using SAS and the alternate interior angle theorem, prove that a line segment joining midpoints of two sides of a triangle is parallel to and half the length of the third side. Apply this theorem to a line segment that cuts two sides of a triangle proportionally. Generalize this theorem to prove that the figure formed by joining consecutive midpoints of sides of an arbitrary quadrilateral is a parallelogram. (This result is known as the Midpoint Quadrilateral Theorem or Varignon's Theorem.) Use cardboard cutouts to illustrate that the altitude to the hypotenuse divides a right triangle into two triangles that are similar to the original triangle. Then use AA to prove this theorem. Then, use this result to establish the Pythagorean relationship among the sides of a right triangle ($a2 + b2 = c2$) and thus obtain an algebraic proof of the Pythagorean Theorem. Prove that the altitude to the hypotenuse of a right triangle is the geometric mean of the two segments into which its foot divides the hypotenuse. Prove the converse of the Pythagorean Theorem, using the theorem itself as one step in the proof. Some students might engage in an exploration of Pythagorean Triples (e.g., 3-4-5, 5-12-13, etc.), which provides an algebraic extension and an opportunity to explore patterns. **Explanations and Examples: G.SRT.4** Use AA, SAS, SSS similarity theorems to prove triangles are similar. Use triangle similarity to prove other theorems about triangles o Prove a line parallel to one side of a triangle divides the other two proportionally, and it's converse o Prove the Pythagorean Theorem using triangle similarity. Students may use geometric simulation software to model transformations and demonstrate a sequence of transformations to show congruence or similarity of figures. **Examples:** · Prove that if two triangles are similar, then the ratio of corresponding attitudes is equal to the ratio of corresponding sides. How does the Pythagorean Theorem support the case for triangle similarity? View the video below and create a visual proving the Pythagorean Theorem using similarity. http://www.youtube.com/watch?v=LrS5_l-gk94 To prove the Pythagorean Theorem using triangle similarity: We can cut a right triangle into two parts by dropping a perpendicular onto the hypotenuse

Title of Unit	Circles and Expressing Geometric Properties through Equations	**Grade Level**	10th grade
Curriculum Area	Geometry	**Time Frame**	6-7 weeks
Developed By	Munira Jamali		
Identify Desired Results (Stage 1)			
Content Standards			

G.C.2. Identify and describe relationships among inscribed angles, radii, and chords. Include the relationship between central, inscribed, and circumscribed angles; inscribed angles on a diameter are right angles; the radius of a circle is perpendicular to the tangent where the radius intersects the circle.
G.C.5 Derive using similarity the fact that the length of the arc intercepted by an angle is proportional to the radius, and define the radian measure of the angle as the constant of proportionality; derive the formula for the area of a sector. Convert between degrees and radians. CA
G.MG.1 Use geometric shapes, their measures, and their properties to describe objects (e.g., modeling a tree trunk or a human torso as a cylinder).*
G.GPE.4 Use coordinates to prove simple geometric theorems algebraically. For example, prove or disprove that a figure defined by four given points in the coordinate plane is a rectangle; prove or disprove that the point (1, 3) lies on the circle centered at the origin and containing the point (0, 2)
G.GPE.5. Prove the slope criteria for parallel and perpendicular lines and use them to solve geometric problems (e.g., find the equation of a line parallel or perpendicular to a given line that passes through a given point)
G.GC.3 Construct the inscribed and circumscribed circles of a triangle, and prove properties of angles for a quadrilateral inscribed in a circle
G.GPE.7 Use coordinates to compute perimeters of polygons and areas of triangles and rectangles, e.g., using the distance formula
G.GPE.1 Derive the equation of a circle of given center and radius using the Pythagorean Theorem; complete the square to find the center and radius of a circle given by an equation.

Understandings	Essential Questions	
Overarching Understanding	**Overarching**	**Topical**
The algebraic representation of a geometric problem can be used to prove theorems in a coordinate plane. • The concept of similarity as it relates to circles can be extended with proof. • Relationships between angles, radii and chords will be investigated. • Similarities will be applied to derive an arc length and a sector area •Justify algebraically the relationships between slopes of parallel and perpendicular lines as they can be established through proof.	How might we use “constant of proportionality” to define radian measure? • How can we write the equation for a circle or parabola? • How can algebraic representation of a geometric problem be used to prove theorems in coordinate plane? • How can the relationships between angles, radii, and cords	What is radian measure How are arc length and area of circle determined by proportionality rule? What is criteria for two line to be parallel or perpendicular What is equation of circle?
Related Misconceptions		

Sectors and segments are often used interchangeably in everyday conversation. Care should be taken to distinguish these two geometric concepts. The formulas for converting radians to degrees and vice versa are easily confused. Knowing that the degree measure of given angle is always a number larger than the radian measure can help students use the correct unit. **Common Misconceptions: G.GPE.1-2** Because new vocabulary is being introduced in this cluster, remembering the names of the conic sections can be problematic for some students. The Euclidean distance formula involves squared, subscripted variables whose differences are added. The notation and multiplicity of steps can be a serious stumbling block for some students. The method of completing the square is a multi-step process that takes time to assimilate. A geometric demonstration of completing the square can be helpful in promoting conceptual understanding. **Common Misconceptions: G.GPE.4-7** Students may claim that a vertical line has infinite slopes. This suggests that infinity is a number. Since applying the slope formula to a vertical line leads to division by zero, we say that the slope of a vertical line is undefined. Also, the slope of a horizontal line is 0. Students often say that the slope of vertical and/or horizontal lines is "no slope," which is incorrect	between angles, radii, and cords be investigated? • What is always true about the slopes of perpendicular (or, parallel) lines, and how can a proof be written to exemplify this?	

Knowledge Students will know…	**Skills** Students will be able to…
To investigate circles and prove theorems about them. Connecting to their prior experience with the coordinate plane, they prove geometric theorems using coordinates and describe shapes with equations.	Understand and apply theorems about circles. Find arc lengths and areas of sectors of circles. Translate between the geometric description and the equation for a conic section

Assessment Evidence (Stage 2)

Performance Task Description	
• **Goal** • **Role** • **Audience** • **Situation** • **Product/Performance** • **Standards**	Illustrative Mathematics: "Right triangles inscribed in circles I" http://www.illustrativemathematics.org/illustrations/1091 Illustrative Mathematics: "Right triangles inscribed in circles II" http://www.illustrativemathematics.org/illustrations/1093 llustrative Mathematics: "Tangent Lines and the Radius of a Circle" http://www.illustrativemathematics.org/illustrations/963 Illustrative Mathematics: "Neglecting the Curvature of the Earth" http://www.illustrativemathematics.org/illustrations/1345 "Equations of Circles 1" – Mathematics Assessment Project This lesson unit is intended to help you assess how well students are able to: •Use the Pythagorean theorem to derive the equation of a circle. •Translate between the geometric features of circles and their equations. http://map.mathshell.org/materials/lessons.php?taskid=406#task406 illustrative Mathematics: "Slopes and Circles" http://www.illustrativemathematics.org/illustrations/479 Illustrative Mathematics: "Explaining the equation for a circle http://www.illustrativemathematics.org/illustrations/1425

Other Evidence

PARCC –
http://www.parcconline.org/samples/ma thematics/high-school-mathematics
http://www.parcconline.org/sites/parcc/files/PARCC_SampleItems_Mathematic s_HSGeoMathIIIGeometricConnection _081913_Final_0.pd

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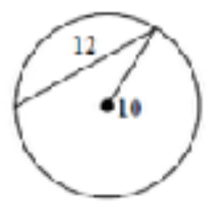
Explanations and Examples: G.C.2

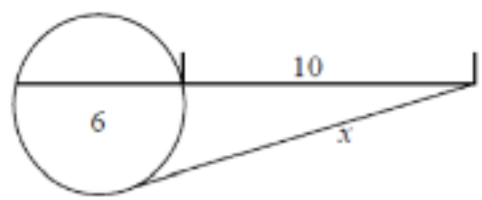
Identify central angles, inscribed angles, circumscribed angles, diameters, radii, chords, and tangents.
Describe the relationship between a central angle and the arc it intercepts.
Describe the relationship between an inscribed angle and the arc it intercepts.
Describe the relationship between a circumscribed angle and the arcs it intercepts.
Recognize that an inscribed angle whose sides intersect the endpoints of the diameter of a circle is a right angle.
Recognize that the radius of a circle is perpendicular to the tangent where the radius intersects the circle.

Examples:

Given the circle below with radius of 10 and chord length of 12, find the distance from the chord to the center of the circle.



Find the unknown length in the picture below.



Solution:

The theorem for a secant segment and a tangent segment that share an endpoint not on the circle states that for the picture below secant segment QR and the tangent segment SR share and endpoint R, not on the circle. Then the length of SR squared is equal to the product of the lengths of QR and KR .