Title of Unit	The Number System	Grade Level	8th grade Level 1 Saturday
Curriculum Area	Mathematics	Time Frame	3-4 weeks
Developed By	Munira Jamali		
Identify Desired Results (Stage 1)			
Content Standards			

Number System 6.NS.1. Interpret and compute quotients of fractions, and solve word problems involving division of fractions by fractions, e.g., by using visual fraction models and equations to represent the problem. For example, create a story context for (2/3) ÷ (3/4) and use a visual fraction model to show the quotient; use the relationship between multiplication and division to explain that (2/3) ÷ (3/4) = 8/9 because 3/4 of 8/9 is 2/3. (In general, (a/b) ÷ (c/d) = ad/bc.)
How much chocolate will each person get if 3 people share 1/2 lb of chocolate equally? How many 3/4-cup servings are in 2/3 of a cup of yogurt? How wide is a rectangular strip of land with length 3/4 mi and area 1/2 square mi?
6.NS.2. Fluently divide multi-digit numbers using the standard algorithm. 6.NS.3. Fluently add, subtract, multiply, and divide multi-digit decimals using the standard algorithm for each operation.
6.NS.4. Find the greatest common factor of two whole numbers less than or equal to 100 and the least common multiple of two whole numbers less than or equal to 12. Use the distributive property to express a sum of two whole numbers 1–100 with a common factor as a multiple of a sum of two whole numbers with no common factor. For example, express 36 + 8 as 4 (9 + 2).
6.NS.5. Understand that positive and negative numbers are used together to describe quantities having opposite directions or values (e.g., temperature above/below zero, elevation above/below sea level, credits/debits, positive/negative electric charge); use positive and negative numbers to represent quantities in real-world contexts, explaining the meaning of 0 in each situation.
6.NS.6. Understand a rational number as a point on the number line. Extend number line diagrams and coordinate axes familiar from previous grades to represent points on the line and in the plane with negative number coordinates.
a. Recognize opposite signs of numbers as indicating locations on opposite sides of 0 on the number line; recognize that the opposite of the opposite of a number is the number itself, e.g.,-(-3) = 3, and that 0 is its own opposite.
b. Understand signs of numbers in ordered pairs as indicating locations in quadrants of the coordinate plane; recognize that when two ordered pairs differ only by signs, the locations of the points are related by reflections across one or both axes. c. Find and position integers and other rational numbers on a horizontal or vertical number line diagram; find and position pairs of integers and other rational numbers on a coordinate plane.
6.NS.7. Understand ordering and absolute value of rational numbers. a. Interpret statements of inequality as statements about the relative position of two numbers on a number line diagram. For example, interpret –3 > –7 as a statement that –3 is located to the right of –7 on a number line oriented from left to right. b. Write, interpret, and explain statements of order for rational numbers in real-world contexts. For example, write –3°C > -7°C to express the fact that –3°C is warmer than -7°C. c. Understand the absolute value of a rational number as its distance from 0 on the number line; interpret absolute value as magnitude for a positive or negative quantity in a real-world situation. For example, for an account balance of –30 dollars, write |–30| = 30 to describe the size of the debt in dollars. d. Distinguish comparisons of absolute value from statements about order. For example, recognize that an account balance less than –30 dollars represents a debt greater than 30 dollars.
6.NS.8. Solve real-world and mathematical problems by graphing points in all four quadrants of the coordinate plane. Include use of coordinates and absolute value to find distances between points with the same first coordinate or the same second coordinate.

Understandings	Essential Questions	
Overarching Understanding	**Overarching**	**Topical**

Procedures used for dividing fractions can be logically explained in several ways. The system of rational numbers includes negative numbers as well as positive ones. · Rational number can be arranged in order. · Absolute value can be described in more than way, depending upon the real-world context. It can be distance, or it can be size (magnitude). · Points can be graphed in all four quadrants of a coordinate grid by using ordered pairs to determine location. · A rational number can be represented as a point on a number line and the number line can be used as a tool to order rational numbers.	How can you compute fractions by using visual fraction models and equations? 2. How do you find the GCF of two whole numbers using the distributive property? 3. How do you use positive and negative numbers to describe quantities having opposite values? 4. What is a rational number and how can you graph it? 5. What is absolute value? 6. How can you apply inverse operations in solving problem	How do number properties assist in computation? Is estimation more appropriate than finding an exact answer? How do we use ordinal numbers in everyday life?
Related Misconceptions		
Students may believe that dividing by 1/2 is the same as dividing in half. Dividing by half means to find how many ½ s there are in a quantity, whereas, dividing in half means to take a quantity and split it into two equal parts. Thus 7 divided by 1/2 = 14 and 7 divided in half equals 3 1/2.		
Knowledge Students will know…	**Skills** Students will be able to…	
Description of the critical area: Students use the meaning of fractions, the meanings of multiplication and division, and the relationship between multiplication and division to understand and explain why the procedures for dividing fractions make sense. Students use these operations to solve problems. Students extend their previous understandings of number and the ordering of numbers to the full system of rational numbers, which includes negative rational numbers, and in particular negative integers. They reason about the order and absolute value of rational numbers and about the location of points in all four quadrants of the coordinate plan	Apply and extend previous understandings of multiplication and division to divide fractions by fractions. Compute fluently with multi-digit numbers and find common factors and multiples. Apply and extend previous understandings of numbers to the system of rational numbers.	
Assessment Evidence (Stage 2)		
Performance Task Description		

• **Goal** • **Role** • **Audience** • **Situation** • **Product/ Performance** • **Standards**	6.NS.6 Fractional Clothesline http://illuminations.nctm.org/Lesson.aspx?id=2867 6.NS.4 The Product Game http://illuminations.nctm.org/LessonDetail.aspx?id=U100 The Venn Factor http://illuminations.nctm.org/LessonDetail.aspx?id=L859 6.NS.5 Zip, Zilch, Zero http://illuminations.nctm.org/LessonDetail.aspx?id=L819
Other Evidence	
PARCC – Fraction Model http://www.parcconline.org/sites/par cc/files/FractionModelFINAL.pdf 6.NS.1 Video explanation of division of fractions Invert and Multiply? Mathematics Assessment Project 6.NS.4 Pedro's Tables 6.NS Interpreting Multiplication and Division Adding and Subtracting Directed Numbers Inside Mathematics Winning Lines Concept Task Fraction of a Fraction http://www.lausd.net/math/InstructionalGuides/Subjects/G6/PDF%20Documents/03.%20Fraction%20of%20a%20Fraction.pdf Linking Fractions http://www.lausd.net/math/InstructionalGuides/Subjects/G6/PDF%20Documents/04.%20Linking%20Fractions.pdf	
Learning Plan (Stage 3)	

• **Where are your students headed? Where have they been? How will you make sure the students know where they are going?** • **How will you hook students at the beginning of the unit?** • **What events will help students experience and explore the big idea and questions in the unit? How will you equip them with needed skills and knowledge?** • **How will you cause students to reflect and rethink? How will you guide them in rehearsing, revising, and refining their work?** • **How will you help students to exhibit and self-evaluate their growing skills, knowledge, and understanding throughout the unit?** • **How will you tailor and otherwise personalize the learning plan to optimize the engagement and effectiveness of ALL students, without compromising the goals of the unit?** • **How will you organize and sequence the learning activities to optimize the engagement and achievement of ALL students?**	**Standard NS .1** Instructional Strategies Computation with fractions is best understood when it builds upon the familiar understandings of whole numbers and is paired with visual representations. Solve a simpler problem with whole numbers, and then use the same steps to solve a fraction divided by a fraction. Looking at the problem through the lens of "How many groups?" or "How many in each group?" helps visualize what is being sought. For example: 12÷3 means; How many groups of three would make 12? Or how many in each of 3 groups would make 12? Thus 7/2 ÷ 1/4 can be solved the same way. How many groups of 1/4 make 7/2 ? Or, how many objects in a group when 7/2 fills one fourth? Creating the picture that represents this problem makes seeing and proving the solutions easier. Set the problem in context and represent the problem with a concrete or pictorial model. 5/4 ÷ 1/2 5/4 cups of nuts fills 1/2 of a container. How many cups of nuts will fill the entire container? Teaching "invert and multiply" without developing an understanding of why it works first leads to confusion as to when to apply the shortcut. Learning how to compute fraction division problems is one part, being able to relate the problems to real-world situations is important. Providing opportunities to create stories for fraction problems or writing equations for situations is needed. **Standard (NS. 2 through 4)** As students study whole numbers in the elementary grades, a foundation is laid in the conceptual understanding of each operation. Discovering and applying multiple strategies for computing creates connections which evolve into the proficient use of standard algorithms. Fluency with an algorithm denotes an ability that is efficient, accurate, appropriate and flexible. Division was introduced in Grade 3 conceptually, as the inverse of multiplication. In Grade 4, division continues using place-value strategies, properties of operations, and the relationship with multiplication, area models, and rectangular arrays to solve problems with one digit divisors. In Grade 6, fluency with the algorithms for division and all operations with decimals is developed. Fluency is something that develops over time; practice should be given over the course of the year as students solve problems related to other mathematical studies. Opportunities to determine when to use paper pencil algorithms, mental math or a computing tool is also a necessary skill and should be provided in problem solving situations. Greatest common factor and least common multiple are usually taught as a means of combining fractions with unlike denominators. This cluster builds upon the previous learning of the multiplicative structure of whole numbers, as well as prime and composite numbers in Grade 4. Although the process is the same, the point is to become aware of the relationships

Title of Unit	**Developing Understanding and Application of Proportional Relationships**				
Curriculum Area	Mathematics	**Time Frame**	3-4 weeks		
Developed By	Munira Jamali				
Identify Desired Results (Stage 1)					
Content Standards					

7.RP.1 Compute unit rates associated with ratios of fractions, including ratios of lengths, areas and other quantities measured in like or different units. For example, if a person walks 1/2 mile in each 1/4 hour, compute the unit rate as the complex fraction 1/2/1/4 miles per hour, equivalently 2 miles per hour.

7.RP.2 Recognize and represent proportional relationships between quantities. a. Decide whether two quantities are in a proportional relationship, e.g., by testing for equivalent ratios in a table or graphing on a coordinate plane and observing whether the graph is a straight line through the origin. b. Identify the constant of proportionality (unit rate) in tables, graphs, equations, diagrams, and verbal descriptions of proportional relationships. c. Represent proportional relationships by equations. For example, if total cost t is proportional to the number n of items purchased at a constant price p, the relationship between the total cost and the number of items can be expressed as t = pn. d. Explain what a point (x, y) on the graph of a proportional relationship means in terms of the situation, with special attention to the points (0, 0) and (1, r) where r is the unit rate.

7.RP.3 Use proportional relationships to solve multistep ratio and percent problems. Examples: simple interest, tax, markups and markdowns, gratuities and commissions, fees, percent increase and decrease, percent error.

Understandings	Essential Questions	
Overarching Understanding	**Overarching**	**Topical**

Proportional reasoning is essential in problem solving · Understanding mathematical relationships allows us to make predictions, calculate and model unknown quantities. · Proportional relationships express how quantities change in relationship to each other. (Look at NYC Common Core Grade 7)	How can proportions be used to solve problems? · When is a relationship proportional? · How can proportions increase our understanding of the real world? · How does the mathematical use of the word similar differ from the everyday use? · How can similarity help us solve measurement	Why are ratios and proportions important?? Does my answer make sense? How can I compare two different things? (How can we show the relationship between two quantities or values?) How should I solve it? (What number form should be used to solve a problem?)
Related Misconceptions		

Lessons may need to be customized if the class period is not long enough to do all of what is presented and/or if students lack prerequisite skills and understanding to move through the entire lesson in the time allotted. A suggestion for customizing the lesson is to first decide upon and designate each question, example, exercise, or challenge as either “Must Do” or “Could Do.” A: Select “Must Do” dialogue, questions, and problems that meet the Student Outcome(s) while still providing a coherent experience for students; reference the ladder. The expectation should be that the majority of the class will be able to complete the “Must Do” portions of the lesson within the allocated time. While choosing the “Must Do” portions of the lesson, keep in mind the need for a balance of dialogue and conceptual questioning, application problems, and abstract problems, and a balance between students using pictorial/graphical representations and abstract representations. Highlight dialogue to be included in the delivery of instruction so that students have a chance to articulate and consolidate understanding as they move through the	problems? · What are the connections between similarity, geometry and algebra	

Knowledge Students will know…	**Skills** Students will be able to…

To extend their understanding of ratios and develop understanding of proportionality to solve single- and multi-step problems. To use their understanding of ratios and proportionality to solve a wide variety of percent problems, including those involving discounts, interest, taxes, tips, and percent increase or decrease. To solve problems about scale drawings by relating corresponding lengths between the objects or by using the fact that relationships of lengths within an object are preserved in similar objects. To graph proportional relationships and understand the unit rate informally as a measure of the steepness of the related line, called the slope. To distinguish proportional relationships from other relationships To reason about relationships among two-dimensional figures using scale drawings and informal geometric constructions, and they gain familiarity with relationships between angles formed by Intersecting lines.	Analyze proportional relationships and use them to solve real-world and mathematical problem Draw, construct, and describe geometrical figures and describe the relationships between them Make sense of problems and persevere in solving them. Reason abstractly and quantitatively To Draw, construct, and describe geometrical figures and describe the relationships between them.

Assessment Evidence (Stage 2)

Performance Task Description

- **Goal**
- **Role**
- **Audience**
- **Situation**
- **Product/ Performance**
- **Standards**

Solving Linear Equations
http://map.mathshell.org/download.php?fileid=1635
This lesson unit is intended to help you assess how well students are able to:
• Form and solve linear equations involving factorizing and using the distributive law. In particular, this unit aims to help you identify and assist students who have difficulties in:
• Using variables to represent quantities in a real-world or mathematical problem. • Solving word problems leading to equations of the form
$px + q = r$ and $p(x + q) = r$.

7.EE: Use properties of operations to generate equivalent expressions. Solve real-life and mathematical problems using numerical and algebraic expressions and equations.

The wheel shop
www.insidemathematics.org/assets/problems-of-the-month/the%20wheel%20shop.pdf

The goal is for all students to have the experience of attacking and solving non–routine problems and developing their mathematical reasoning skills. Although obtaining and justifying solutions to the problems is the objective, the process of learning to problem–solve is even more important
7.EE.2. Understand that rewriting an expression in different forms in a problem context can shed light on the problem and how the quantities in it are related. For example, $a + 0.05a = 1.05a$ means that "increase by 5%" is the same as "multiply by 1.05
7.EE.4. Use variables to represent quantities in a real-world or mathematical problem, and construct simple equations and inequalities to solve problems by reasoning about the quantities.

Other Evidence

Illustrative Mathematics

• 7.EE.1– The Mango Problem
http://illuminations.nctm.org/Lesson.aspx?id=1037

• 7.EE.1– The Sailor and Cocoanut Problem
http://illuminations.nctm.org/404.aspx?aspxerrorpath=/lessons/6-8/mangoes/Classic-AS-Sailor.pdf

• 7.EE.1 and 7.EE.2 Pan Balance - Expressions
http://illuminations.nctm.org/Lesson.aspx?id=2747

• 7.EE.1 – Miles to Kilometers
http://illustrativemathematics.org/illustrations/433

• 7 EE.3 – Discounted Books
http://illustrativemathematics.org/illustrations/478

• 7.EE.4 and 4b. – Fishing Adventures 2
https://www.illustrativemathematics.org/content-standards/tasks/643

• 7 EE.4b – Sport Equipment Se

Learning Plan (Stage 3)

• **Where are your students headed? Where have they been? How will you make sure the students know where they are going?** • **How will you hook students at the beginning of the unit?** • **What events will help students experience and explore the big idea and questions in the unit? How will you equip them with**	**7.RP.1** Students continue to work with unit rates from 6th grade; however, the comparison now includes fractions compared to fractions. For example, if 1/2 gallon of paint covers 1/6 of a wall, then the amount of paint needed for the entire wall can be computed by 1/2 gal divided by 1/6 wall. This calculation gives 3 gallons. This standard requires only the use of ratios as fractions. Fractions may be proper or improper. Building from the development of rate and unit concepts in Grade 6, applications now need to focus on solving unit-rate problems with more sophisticated numbers: fractions per fractions. Proportional relationships are further developed through the analysis of graphs, tables, equations and diagrams. Ratio tables serve a valuable purpose in the solution of proportional problems. This is the time to push for a deep understanding of what a representation of a proportional relationship looks like and what the characteristics are: a straight line through the origin on a graph, a "rule" that applies for all ordered pairs, an equivalent ratio or an expression that describes the situation, etc. This is not the time for students to learn to cross multiply to solve problems. Because percents have been introduced as rates in Grade 6, the work with percents should continue to follow the thinking involved with rates and proportions. Solutions to problems can be found by using the same strategies for solving rates, such as looking for equivalent ratios or based upon understandings of decimals. Previously, percents have focused on "out of 100"; now percents above 100 are encountered. Providing opportunities to solve problems based within contexts that are relevant to seventh graders will connect meaning to rates, ratios and proportions. Examples include: researching newspaper ads and constructing their own question(s), keeping a log of prices (particularly sales) and determining savings by purchasing items on sale, timing students as they walk a lap on the track and figuring their rates, creating open-ended

Title of Unit	Understand Expressions and Equations	**Grade Level**	Level 1 Saturdayl
Curriculum Area	Mathematics	**Time Frame**	3-5 weeks
Developed By	Munira Jamali		
Identify Desired Results (Stage 1)			
Content Standards			

7.EE.1.
Apply properties of operations as strategies to add, subtract, factor, and expand linear expressions with rational coefficients
7.EE.2.
Understand that rewriting an expression in different forms in a problem context can shed light on the problem and how the quantities in it are related. For example, a + 0.05a = 1.05a means that "increase by 5%" is the same as "multiply by 1.05."
7.EE.3.
Solve multi-step real-life and mathematical problems posed with positive and negative rational numbers in any form (whole numbers, fractions, and decimals), using tools strategically. Apply properties of operations to calculate with numbers in any form; convert between forms as appropriate; and assess the reasonableness of answers using mental computation and estimation strategies. For example: If a woman making $25 an hour gets a 10% raise, she will make an additional 1/10 of her salary an hour, or $2.50, for a new salary of $27.50. If you want to place a towel bar 9 3/4 inches long in the center of a door that is 27 1/2 inches wide, you will need to place the bar about 9 inches from each edge; this estimate can be used as a check on the exact computation.
7.EE.4.
Use variables to represent quantities in a real-world or mathematical problem, and construct simple equations and inequalities to solve problems by reasoning about the quantities. a. Solve word problems leading to equations of the form px + q = r and p(x + q) = r, where p, q, and r are specific rational numbers. Solve equations of these forms fluently. Compare an algebraic solution to an arithmetic solution, identifying the sequence of the operations used in each approach. For example, the perimeter of a rectangle is 54 cm. Its length is 6 cm. What is its width? b. Solve word problems leading to inequalities of the form px + q > r or px + q < r, where p, q, and r are specific rational numbers. Graph the solution set of the inequality and interpret it in the context of the problem. For example: As a salesperson, you are paid $50 per week plus $3 per sale. This week you want your pay to be at least $100. Written inequality for the number of sales you need to make, and describe the solutions

<table>
<tr><th colspan="2">Understandings</th><th colspan="2">Essential Questions</th></tr>
<tr><th colspan="2">Overarching Understanding</th><th>Overarching</th><th>Topical</th></tr>
<tr><td colspan="2">Generating equivalent, linear expressions with rational coefficients using the properties of operations will lead to solving linear equation.
· Discovering that rewriting expressions in different forms in a problem context leads to understanding that the values are equivalent.
· Ability to solve and explain real life and mathematical problems involving rational numbers using numerical and algebraic expressions is important for preparation for HS Algebra. ·
Constructing simple equations and inequalities to solve real life word problems is a necessary concept.</td><td rowspan="2">How can I apply the order of operations and the fundamentals of algebra to solve problems?
How can I justify that multiple representations in the context of a problem are equivalent expressions?
How do I assess the reasonableness of my answer?
How will I use the properties of equality to explain the order of the steps in solving equations and</td><td rowspan="2">Do mathematical models conceal as much as they reveal?

What patterns or relationships do we see in each type of mathematics?

What are the different ways to represent the patterns or relationships?

What different interpretations can</td></tr>
<tr><th colspan="2">Related Misconceptions</th></tr>
</table>

As students begin to build and work with expressions containing more than two operations, students tend to set aside the order of operations. For example having a student simplify an expression like 8 + 4(2x - 5) + 3x can bring to light several misconceptions. Do the students immediately add the 8 and 4 before distributing the 4? Do they only multiply the 4 and the 2x and not distribute the 4 to both terms in the parenthesis? Do they collect all like terms 8 + 4 – 5, and 2x + 3x? Each of these show gaps in students' understanding of how to simplify numerical expressions with multiple operations.	equations and inequalities? How do I interpret the solutions for equations and inequalities in the context of the problem?	interpretations can be obtained from a particular pattern or relationship? What predictions can the patterns or relationships support? How can we use or test our predictions? Are they valid? Are they significant? Where in the real world would I find patterns?
Knowledge Students will know…	**Skills** Students will be able to…	
To understand equivalent expressions as they apply the properties of operations (associative, commutative, and distributive) to write expressions in both standard form (by expanding products into sums) and in factored form (by expanding sums into products). To use linear equations to solve unknown angle problems and other problems presented within context to understand that solving algebraic equations is all about the numbers. It is assumed that a number already exists to satisfy the equation and context; we just need to discover it. A number sentence is an equation that is said to be true if both numerical expressions evaluate to the same number; it is said to be false otherwise. To use the number line to understand the properties of inequality and recognize when to preserve the inequality and when to reverse the inequality when solving problems leading to inequalities.	● Use properties of operations to generate equivalent expressions ● Solve real-life and mathematical problems using numerical and algebraic expressions and equations ● Solve real-life and mathematical problems involving angle measure, area, surface area, and volume ● Use properties of operations to generate equivalent expressions ● Solve real-life and mathematical problems using numerical and algebraic expressions and equations	
Assessment Evidence (Stage 2)		
Performance Task Description		

• **Goal** • **Role** • **Audience** • **Situation** • **Product/ Performance** • **Standards**	**7.RP.1 and 7.RP.2 Proportion and Non-proportion Situations** http://map.mathshell.org/materials/lessons.php?taskid=483#task483 This lesson unit is intended to help you assess whether students are able to: • Identify when two quantities vary in direct proportion to each other. • Distinguish between direct proportion and other functional relationships. • Solve proportionality problems using efficient methods Standard 7.RP: Analyze proportional relationships and use them to solve real-world and mathematical problems **7.RP.1 and 7.G.1 Developing a Sense of Scale** http://map.mathshell.org/download.php?fileid=1631 This lesson unit is intended to help you assess whether students recognize relationships of direct proportion and how well they solve problems that involve proportional reasoning. In particular, it is intended to help you identify those students who: • Use inappropriate additive strategies in scaling problems, which have a multiplicative structure. • Rely on piecemeal and inefficient strategies such as doubling, halving, and decomposition and have not developed a single multiplier strategy for solving proportionality problems. • See multiplication as making numbers bigger and division as making numbers smaller 7.RP: Analyze proportional relationships and use them to solve real-world and mathematical problems **7.RP.3 Increasing and Decreasing Quantities by a Percent** http://map.mathshell.org/lessons.php?unit=7100&collection=8&redir=1 This lesson unit is intended to help you assess how well students are able to interpret percent increase and decrease and in particular, to identify and help students who have the following difficulties: • Translating between percents, decimals, and fractions. • Representing percent increase and decrease as multiplication. • Recognizing the relationship between increases and decrease 7.RP: Analyze proportional relationships and use them to solve real-world and mathematical problems. 7.NS: Apply and extend previous understandings of operations with fractions. 7.EE: Use properties of operations to generate equivalent expressions Drawing to Scale: A Garden http://map.mathshell.org/download.php?fileid=1641 This lesson unit is intended to help assess how well students are able to interpret and use scale drawings to plan a garden layout. This involves using proportional reasoning and metric units. Standards Mathematical Content in the Common Core State Standards for Mathematics: 7.G: Draw, construct, and describe geometrical figures and describe the relationships between them. Solve real-life and mathematical problems involving angle measure, area, surface area, and volume. 7.EE: Solve real-life and mathematical problems using numerical and algebraic expressions and equations

Other Evidence
Illustrative Mathematics 7.RP.1 Molly's Run www.illustrativemathematics.org/content-standards/tasks/828 7.RP.2 Music Companies, Variations 1 https://www.illustrativemathematics.org/content-standards/tasks/95 7.RP.1 Cooking with Whole Cup https://www.illustrativemathematics.org/content-standards/tasks/470 • 7.RP.1 Track Practice https://www.illustrativemathematics.org/content-standards/tasks/82 • 7 RP.2 Art Class, Variations 1& Variations 2 - Buying Coffee https://www.illustrativemathematics.org/content-standards/tasks/104 7.RP.2d Robot Races 7.RP.2 Sore Throats – Variation https://www.illustrativemathematics.org/content-standards/tasks/181
Learning Plan (Stage 3)

- **Where are your students headed? Where have they been? How will you make sure the students know where they are going?**
- **How will you hook students at the beginning of the unit?**
- **What events will help students experience and explore the big idea and questions in the unit? How will you equip them with needed skills and knowledge?**
- **How will you cause students to reflect and rethink? How will you guide them in rehearsing, revising, and refining their work?**
- **How will you help students to exhibit and self-evaluate their growing skills, knowledge, and understanding throughout the unit?**
- **How will you tailor and otherwise personalize the learning plan to optimize the engagement and effectiveness of ALL students, without compromising the goals of the unit?**
- **How will you organize and sequence the learning activities to optimize the engagement and achievement of ALL students?**

7.EE.1.
Apply properties of operations as strategies to add, subtract, factor, and expand linear expressions with rational coefficients
Have students build on their understanding of order of operations and use the properties of operations to rewrite equivalent numerical expressions that were developed in Grade 6. Students continue to use properties that were initially used with whole numbers and now develop the understanding that properties hold for integers, rational and real numbers.
Provide opportunities to build upon this experience of writing expressions using variables to represent situations and use the properties of operations to generate equivalent expressions.
These expressions may look different and use different numbers, but the values of the expressions are the same.
Provide opportunities for students to experience expressions for amounts of increase and decrease. In Standard 2, the expression is rewritten and the variable has a different coefficient. In context, the coefficient aids in the understanding of the situation. Another example is this situation which represents a 10% decrease: b - 0.10b = 1.00b - 0.10b which equals 0.90b or
90% of the amount.
One method that students can use to become convinced that expressions are equivalent is by substituting a numerical value for the variable and evaluating the expression. For example 5(3 +2x) is equal to 5 •3 + 5 • 2x Let x = 6 and substitute 6 for x in both equations.

5(3 + 2•6)	5•3 + 5•2•6
5(3 + 12)	15 + 60
5(15)	75
75	

Provide opportunities for students to use and understand the properties of operations. These include: the commutative, associative, identity, and inverse properties of addition and of multiplication, and the zero property of multiplication. Another method students can use to become convinced that expressions are equivalent is to justify each step of simplification of an expression with an operation property.
Explanations and Examples
7.EE.1 This is a continuation of work from 6th grade using properties of operations and combining like terms. Students apply properties of operations and work with rational
numbers (integers and positive / negative fractions and decimals) to write equivalent expressions.

Title of Unit	Understanding Geometry	Grade Level	Grade 7 Supplemental
Curriculum Area	Mathematics	Time Frame	5-6 weks
Developed By	Munira Jamali		

Identify Desired Results (Stage 1)

Content Standards

7.G.1. Solve problems involving scale drawings of geometric figures, including computing actual lengths and areas from a scale drawing and reproducing a scale drawing at a different scale.
7.G.2. Draw (freehand, with ruler and protractor, and with technology) geometric shapes with given conditions. Focus on constructing triangles from three measures of angles or sides, noticing when the conditions determine a unique triangle, more than one triangle, or no triangle.
7.G.3. Describe the two-dimensional figures that result from slicing three-dimensional figures, as in plane sections of right rectangular prisms and right rectangular pyramids.
7.G.3.1 Describe how two or more objects are related in space (e.g., skew lines, the possible ways three planes might intersect).CA (s/a)2 Solve real-life and mathematical problems involving angle measure, area, surface area, and volume.
7.G.4. Know the formulas for the area and circumference of a circle and use them to solve problems; give an informal derivation of the relationship between the circumference and area of a circle
7.G.5. Use facts about supplementary, complementary, vertical, and adjacent angles in a multistep problem to write and solve simple equations for an unknown angle in a figure.
7.G.6. Solve real-world and mathematical problems involving area, volume and surface area of two- and three dimensional objects composed of triangles, quadrilaterals, polygons, cubes, and right prisms.

Understandings	Essential Questions

Overarching Understanding	Overarching	Topical
Students delve further into several geometry topics they have been developing over the years. Grade 7 presents some of these topics, (e.g., angles, area, surface area, and volume) in the most challenging form students have experienced yet. This unit assumes students understand the basics. The goal is to build a fluency in these difficult problems. The remaining topics, (i.e., working on constructing triangles and taking slices (or cross-sections) of three-dimensional figures) are new to students.	What 2-D figure results from slicing 3-D figures? (cones, spheres, or cylinders) How do you find the surface area and volume of a 3D figure? What is the total number of degrees in supplementary and complementary angles? What is the relationship between vertical and adjacent angles? How would the volume and surface area be affected when dimensions of a figure are doubled and/or triple?	Where in the real world can I find shapes? Where would you find symmetry? How can objects be represented and compared using geometric attributes? Is geometry more like map-making and using a map, or inventing and playing games like chess? How can I identify and describe solid figures by describing the faces, edges, and sides?
Related Misconceptions		
Student's may have misconceptions about correctly setting up proportions, how to read a ruler, doubling side measures, and does not double perimeter. Students may believe: Pi is an exact number rather than understanding that 3.14 is just an approximation of pi. Many students are confused when dealing with circumference (linear measurement) and area. This confusion is about an attribute that is measured using linear units (surrounding) vs. an attribute that is measured using area units (covering).		
Knowledge Students will know…	**Skills** Students will be able to…	

To continue their work with area from Grade 6, solving problems Involving the area and circumference of a circle and surface area of three-dimensional objects. To reason about relationships among two-dimensional figures using scale drawings and informal geometric constructions, and they gain familiarity with relationships between angles formed by Intersecting lines. To work with three-dimensional figures, relating them to two-dimensional figures by examining cross-sections. To solve real-world and mathematical problems involving area, surface area, and volume of two- and three-dimensional objects composed of triangles, quadrilaterals, polygons, cubes, and right prisms.	Draw, construct, and describe geometrical figures and describe the relationships between them Solve real-life and mathematical problems involving angle measure, area, surface area, and volume.

Assessment Evidence (Stage 2)

Performance Task Description

• **Goal** • **Role** • **Audience** • **Situation** • **Product/ Performance** • **Standards**	**7.G.6 Maximizing Area: Gold Rush** http://map.mathshell.org/download.php?fileid=1637 Maximizing Area: Gold Rush This lesson unit is intended to help you assess how well students are able to: • Interpret a situation and represent the variables mathematically. • Select appropriate mathematical methods to use and communicate their reasoning clearly. • Explore the effects on a rectangle's area of systematically varying the dimensions whilst keeping the perimeter constant. Interpret and evaluate the data generated, identifying the optimum case **Standards** 7.G: Draw construct, and describe geometrical figures and describe the relationships between them. Solve real-life and mathematical problems involving angle measure, area, surface area, and volume. 7.EE: Solve real-life and mathematical problems using numerical and algebraic expressions and equations. **Using Dimensions: Designing a Sports Bag** http://moodle.wbrschools.net/pluginfile.php/3902/mod_resource/content/1/using_dimensions_designing_a_sports_bag_complete.pdf This lesson unit is intended to help you assess how well students are able to: • Recognize and use common 2D representations of 3D objects. • Identify and use the appropriate formula for finding the circumference of a circle. **Standards** 7.G: Draw, construct and describe geometrical figures and describe the relationships between them. Solve real-life and mathematical problems involving angle measure, area, surface area, and volume. Drawing to Scale: A Garden http://map.mathshell.org/download.php?fileid=1641 This lesson unit is intended to help assess how well students are able to interpret and use scale drawings to plan a garden layout. This involves using proportional reasoning and metric units. Standards 7.G: Draw, construct, and describe geometrical figures and describe the relationships between them. Solve real-life and mathematical problems involving angle measure, area, surface area, and volume. 7.EE: Solve real-life and mathematical problems using numerical and algebraic expressions and equations. 7.RP: Analyze proportional relationships and use them to solve real-world and mathematical problems. **7.G.6 Estimations and Approximations: The Money Munchers** http://map.mathshell.org/materials/lessons.php?taskid=220#task220 This lesson unit is intended to help you assess how well students are able to: • Model a situation. • Make sensible, realistic assumptions and estimates. • Use assumptions and estimates to create a chain of reasoning, in order to solve a practical problem. 7.G: Solve real-life and mathematical problems involving angle measure, area, surface area, and volume
Other Evidence	

Illustrative Mathematics
7.SP Estimating the Mean State Area
https://www.illustrativemathematics.org/illustrations/260
Engage NY Common Core Curriculum
Module 6 – Geometry

Learning Plan (Stage 3)

• **Where are your students headed? Where have they been? How will you make sure the students know where they are going?** • **How will you hook students at the beginning of the unit?** • **What events will help students experience and explore the big idea and questions in the unit? How will you equip them with needed skills and knowledge?** • **How will you cause students to reflect and rethink? How will you guide them in rehearsing, revising, and refining their work?** • **How will you help students to exhibit and self-evaluate their growing skills, knowledge, and understanding throughout the unit?** • **How will you tailor and otherwise personalize the learning plan to optimize the engagement and effectiveness of ALL students, without compromising the goals of the unit?** • **How will you organize and sequence the learning activities to optimize the engagement and achievement of ALL students?**	**7.G.1** This cluster focuses on the importance of visualization in the understanding of Geometry. Being able to visualize and then represent geometric figures on paper is essential to solving geometric problems. Scale drawings of geometric figures connect understandings of proportionality to geometry and lead to future work in similarity and congruence. As an introduction to scale drawings in geometry, students should be given the opportunity to explore scale factor as the number of time you multiple the measure of one object to obtain the measure of a similar object. It is important that students first experience this concept concretely progressing to abstract contextual situations. Pattern blocks (not the hexagon) provide a convenient means of developing the foundation of scale. Choosing one of the pattern blocks as an original shape, students can then create the next-size shape using only those same-shaped blocks. Questions about the relationship of the original block to the created shape should be asked and recorded. A sample of a recording sheet is shown. This can be repeated for multiple iterations of each shape by comparing each side length to the original's side length. An extension would be for students to compare the later iterations to the previous. Students should also be expected to use side lengths equal to fractional and decimal parts. In other words, if the original side can be stated to represent 2.5 inches, what would be the 34 new lengths and what would be the scale? Provide opportunities for students to use scale drawings of geometric figures with a given scale that requires them to draw and label the dimensions of the new shape. Initially, measurements should be in whole numbers, progressing to measurements expressed with rational numbers. This will challenge students to apply their understanding of fractions and decimals. After students have explored multiple iterations with a couple of shapes, ask them to choose and replicate a shape with given scales to find the new side lengths, as well as both the perimeters and areas. Starting with simple shapes and whole-

Title of Unit	Statistics and Probability	**Grade Level**	8th grade Level 1 Saturdayl
Curriculum Area	Mathematics	**Time Frame**	5-6 weeks
Developed By	Munira Jamali		

Identify Desired Results (Stage 1)

Content Standards

7.SP.5. Understand that the probability of a chance event is a number between 0 and 1 that expresses the likelihood of the event occurring. Larger numbers indicate greater likelihood. A probability near 0 indicates an unlikely event, a probability around 1/2 indicates an event that is neither unlikely nor likely, and a probability near 1 indicates a likely event.
7.SP.6. Approximate the probability of a chance event by collecting data on the chance process that produces it and observing its long-run relative frequency, and predict the approximate relative frequency given the probability. For example, when rolling a number cube 600 times, predict that a 3 or 6 would be rolled roughly 200 times, but probably not exactly 200 times.
7.SP.7. Develop a probability model and use it to find probabilities of events. Compare probabilities from a model to observed frequencies; if the agreement is not good, explain possible sources of the discrepancy. a. Develop a uniform probability model by assigning equal probability to all outcomes, and use the model to determine probabilities of events. For example, if a student is selected at random from a class, find the probability that Jane will be selected and the probability that a girl will be selected. b. Develop a probability model (which may not be uniform) by observing frequencies in data generated from a chance process. For example, find the approximate probability that a spinning penny will land heads up or that a tossed paper cup will land open-end down. Do the outcomes for the spinning penny appear to be equally likely based on the observed frequencies?
7.SP.8. Find probabilities of compound events using organized lists, tables, tree diagrams, and simulation.
a. Understand that, just as with simple events, the probability of a compound event is the fraction of outcomes in the sample space for which the compound event occurs. b. Represent sample spaces for compound events using methods such as organized lists, tables and tree diagrams. For an event described in everyday language (e.g., "rolling double sixes"), identify the outcomes in the sample space which compose the event. c. Design and use a simulation to generate frequencies for compound events. For example, use random digits as a simulation tool to approximate the answer to the question: If 40% of donors have type A blood, what is the probability that it will take at least 4 donors to find one with type A blood?

Understandings	Essential Questions

Overarching Understanding	Overarching	Topical
In this unit students begin their study of probability, learning how to interpret probabilities and how to compute probabilities in simple settings. They also learn how to estimate probabilities empirically. Probability provides a foundation for the inferential reasoning developed in the second half of this module. Students build on their knowledge of data distributions that they studied in Grade 6, compare data distributions of two or more populations, and are introduced to the idea of drawing informal inferences based on data from random samples	Probability and Statistics How do you determine which measures of variability should be used to draw informal comparative inferences? How are lists, tables, tree diagrams or simulation used to find the probability of an event? How is probability used to predict frequency of an event?	
Related Misconceptions		

Students may believe: One random sample is not representative of the entire population. Many samples must be taken in order to make an inference that is valid. By comparing the results of one random sample with the results of multiple random samples, students can correct this misconception Students often expect the theoretical and experimental probabilities of the same data to match. By providing multiple opportunities for students to experience simulations of situations in order to find and compare the experimental probability to the theoretical probability, students discover that rarely are those probabilities the same. Students often expect that simulations will result in all of the possibilities. All possibilities may occur in a simulation, but not necessarily. Theoretical probability does use all possibilities. Note examples in simulations when some possibilities are not shown Students often expect the theoretical and experimental probabilities of the same data to match. By providing multiple opportunities for students to experience simulations of situations in order to find and compare the experimental probability to the theoretical probability, students discover that rarely are those probabilities the same. Students often expect that simulations will result in all of the possibilities. All possibilities may occur in a simulation, but not necessarily. Theoretical probability does use all possibilities. Note examples in simulations when some possibilities are not shown		
Knowledge Students will know…	**Skills** Students will be able to…	

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Assessment Evidence (Stage 2)

Performance Task Description

• **Goal** • **Role** • **Audience** • **Situation** • **Product/ Performance** • **Standards**	7.SP.1 Estimating: Counting Tree http://map.mathshell.org/materials/lessons.php?taskid=422#task422 This lesson unit is intended to help you assess how well students are able to: • Solve simple problems involving ratio and direct proportion. • Choose an appropriate sampling method. • Collect discrete data and record them using a frequency table. Standards 7.RP: Analyze proportional relationships and use them to solve real-world and mathematical problems. 7.G: Solve real-life and mathematical problems involving angle measure, area, surface area, and volume. 7.SP: Use random sampling to draw inferences about a population. 7.SP.5-8 Evaluating Statements About Probability http://map.mathshell.org/materials/lessons.php?taskid=220#task220 This lesson unit addresses common misconceptions relating to the probability of simple and compound events. The lesson will help you assess how well students understand concepts of: • Equally likely events. • Randomness. • Sample size.

Other Evidence

7.SP.2 and &.SP.7 Election Poll, Variation 1
http://www.illustrativemathematics.org/illustrations/235

• 7.SP.2 and SP.2 Election Poll, Variation 2
http://www.illustrativemathematics.org/illustrations/559/

Learning Plan (Stage 3)

- **Where are your students headed? Where have they been? How will you make sure the students know where they are going?**
- **How will you hook students at the beginning of the unit?**
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- **How will you cause students to reflect and rethink? How will you guide them in rehearsing, revising, and refining their work?**
- **How will you help students to exhibit and self-evaluate their growing skills, knowledge, and understanding throughout the unit?**
- **How will you tailor and otherwise personalize the learning plan to optimize the engagement and effectiveness of ALL students, without compromising the goals of the unit?**
- **How will you organize and sequence the learning activities to optimize the engagement and achievement of ALL students?**

7.SP.1.

Instructional Strategies

In Grade 6, students used measures of center and variability to describe data. Students continue to use this knowledge in Grade 7 as they use random samples to make predictions about an entire population and
judge the possible discrepancies of the predictions. Providing opportunities for students to use real-life situations from science and social studies shows the purpose for using random sampling to make inferences
about a population.
Make available to students the tools needed to develop the skills and understandings required to produce a
representative sample of the general population. One key element of a representative sample is understanding that a random sampling guarantees that each element of the population has an equal opportunity to be selected in the sample. Have students compare the random sample to population, asking
questions like “Are all the elements of the entire population represented in the sample?” and “Are the
elements represented proportionally?” Students can then continue the process of analysis by determining the measures of center and variability to make inferences about the general population based on the
analysis.
Provide students with random samples from a population, including the statistical measures. Ask students
guiding questions to help them make inferences from the sample.

Explanations and Examples:

7.SP.1 Students recognize that it is difficult to gather statistics on an entire population. Instead a random
sample can be representative of the total population and will generate valid results. Students use this information to draw inferences from data. A random sample must be used in conjunction with the population to get accuracy. For example, a random sample of elementary students cannot be used to give
a survey about the prom.
Example:
The school food service wants to increase the number of students who eat hot lunch in the cafeteria.
The student council has been asked to conduct a survey of the student body to determine the students’ preferences for hot lunch. They have determined two ways to do the survey. The two methods are listed below. Identify the type of