<table>
<tr><th>Title of Unit</th><td>Rational Numbers and Properties of Exponents</td><th>Grade Level</th><td colspan="2">8th Grade Level 3<br>Saturdays</td></tr>
<tr><th>Curriculum Area</th><td>Mathematics</td><th>Time Frame</th><td colspan="2">2-3 weeks</td></tr>
<tr><th>Developed By</th><td colspan="4">Munira Jamali</td></tr>
<tr><th colspan="5">Identify Desired Results (Stage 1)</th></tr>
<tr><th colspan="5">Content Standards</th></tr>
<tr><td colspan="5">• 8.NS.1:<br>Know that numbers that are not rational are called irrational. Understand informally that every number has a decimal expansion; for rational numbers show that the decimal expansion that terminates in the 0s or eventually repeat.<br>• 8.NS.2<br>Use rational approximation of irrational numbers to compare the size of irrational numbers. Locate them approximately on a number line diagram, and estimate the value of expressions (eg, $\pi^2$)<br>• 8.EE.1<br>Know and apply the properties of integer exponents to generate equivalent numerical expressions<br>For example $3^2 x\ 3^{-5} = 3^{-3} = 1/3^2 = 1/27$<br>• 8.EE.3<br>Use numbers expressed in the form of a single digit items a whole number power of 10 to estimate very large or very small quantities and to express how many times as much as one is than the other<br>• 8.EE.4<br>Perform operations with numbers expressed in scientific notation, including problems where both decimal and scientific notation are used. Use scientific notation and choose units of appropriate size for measurements of very large or very small quantities</td></tr>
<tr><th colspan="2">Understandings</th><th colspan="3">Essential Questions</th></tr>
<tr><th colspan="2">Overarching Understanding</th><th colspan="2">Overarching</th><th>Topical</th></tr>
</table>

| | | |
|---|---|---|
| • Students approximate irrational numbers using their understanding of square and cube roots.<br>• Students extend their understanding of the number system by investigating the relationship between the sides of a right triangle.<br>• Students create equivalent expressions using integer exponents.<br>• Students apply their understanding of exponents to express and compare numbers.<br>• Students understand irrational numbers and when to use them in solving problems. | How are numbers rational and irrational numbers related?<br>How do we determine whether two expressions involving exponents are equivalent?<br>How can we express very small or very large numbers using exponential (scientific) notation?<br>How can you investigate the relationships between rational and irrational numbers? | How do I determine the best numerical representation (pictorial, symbolic, objects) for a given situation?<br>• How does finding the common characteristics among similar problems help me to be a more efficient problem solver?<br>• What kinds of experiences help develop number sense? |
| **Related Misconceptions** | | |
| Some students are surprised that the decimal representation of pi does not repeat. Some students believe that if only we keep looking at digits farther and farther to the right, eventually a pattern will emerge.<br>A few irrational numbers are given special names (pi and e), and much attention is given to $\sqrt{2}$. Because we name so few irrational numbers, students sometimes conclude that irrational numbers are unusual and rare. In fact, irrational numbers are much more plentiful than rational numbers, in the sense that they are —denser in the real line.<br>Students may think that the number line only has the numbers that are labeled.<br>Students may confuse the radical sign with the division sign. Students may forget that each rational number has a negative square root, as well as a principal (positive) square root. | | |
| **Knowledge**<br>Students will know… | **Skills**<br>Students will be able to… | |

| | |
|---|---|
| That there are numbers that are not rational, and approximate them by rational numbers<br>Work with radical and integer exponents<br>To perform with numbers expressed in scientific notation including problems where both decimal and scientific notation are used | Approximate irrational numbers using their understanding of square and cube roots<br>Create equivalent expressions using integer exponents<br>Apply understanding of exponents to express and compare numbers<br>Understand irrational numbers and when to use them in solving problems |

## Assessment Evidence (Stage 2)

### Performance Task Description

| | |
|---|---|
| • **Goal**<br>• **Role**<br>• **Audience**<br>• **Situation**<br>• **Product/ Performance**<br>• **Standards** | 8.NS Estimating Square Roots<br>8-NS Calculating and Rounding Numbers<br>N-RN, 8-NS Calculating the square root of 2<br>8.NS Converting Decimal Representations of Rational Numbers to Fraction Representations<br>8.NS Identifying Rational Numbers<br>8.NS Converting Repeating Decimals to Fractions<br>8.NS Comparing Rational and Irrational Numbers<br>8.NS Irrational Numbers on the Number Line<br>8.NS Placing a square root on the number line |

### Other Evidence

http://commoncoretools.me/wp-content/uploads/2013/07/ccssm_progression_NS+Number_2013-07-09.pdf

## Learning Plan (Stage 3)

- **Where are your students headed? Where have they been? How will you make sure the students know where they are going?**
- **How will you hook students at the beginning of the unit?**
- **What events will help students experience and explore the big idea and questions in the unit? How will you equip them with needed skills and knowledge?**
- **How will you cause students to reflect and rethink? How will you guide them in rehearsing, revising, and refining their work?**
- **How will you help students to exhibit and self-evaluate their growing skills, knowledge, and understanding throughout the unit?**
- **How will you tailor and otherwise personalize the learning plan to optimize the engagement and effectiveness of ALL students, without compromising the goals of the unit?**
- **How will you organize and sequence the learning activities to optimize the engagement and achievement of ALL students?**

**Grade 8.NS.1**

**Explanations and Examples:**

Students distinguish between rational and irrational numbers. Any number that can be expressed as a fraction is a rational number. Students recognize that the decimal equivalent of a fraction will either terminate or repeat. Fractions that terminate will have denominators containing only prime factors of 2 and/or 5. This understanding builds on work in 7th grade when students used long division to distinguish between repeating and terminating decimals. Students convert repeating decimals into their fraction equivalent using patterns or algebraic reasoning. One method to find the fraction equivalent to a repeating decimal is shown below.

Change $0.\overline{4}$ to a fraction

· Let $x = 0.4444444 \ldots$

· Multiply both sides so that the repeating digits will be in front of the decimal. In this example, one digit repeats so both sides are multiplied by 10, giving $10x=4.4444444 \ldots$

· Subtract the original equation from the new equation.

$10x = 4.4444444 \ldots$

$x = 0.44444 \ldots$

$9x = 4$

Solve the equation to determine the equivalent fraction.

$$\frac{9x}{9} = \frac{4}{9}$$

$$x = \frac{4}{9}$$

· Solve the equation to determine the equivalent fraction.

Additionally, students can investigate repeating patterns that occur when fractions have a denominator of 9, 99, or 11.

For example is equivalent to is equivalent to

0.5 etc.

A student made the following conjecture and found two examples to support the conjecture.

*If a rational number is not an integer, then the square root of the rational number Is irrational. For example,* $\sqrt{3.6}$ *is irrational and* $\sqrt{1/2}$ is irrational.

Provide two examples of non-integer rational numbers that show that the conjecture is **false.**

*Sample Response*:

· Example 1: 2.25

· Example 2: $1/4$

Students can use graphic organizers to show the relationship between the subsets of the real number

<table>
<tr><th>Title of Unit</th><td>Congruence and Similarity and Problem Solving</td><th>Grade Level</th><td>8th Grade Level 2</td></tr>
<tr><th>Curriculum Area</th><td>Mathematics</td><th>Time Frame</th><td>4-5 weeks</td></tr>
<tr><th>Developed By</th><td colspan="3">Munira Jamali</td></tr>
<tr><th colspan="4">Identify Desired Results (Stage 1)</th></tr>
<tr><th colspan="4">Content Standards</th></tr>
</table>

- **8**.G.1 Verify experimentally the properties of rotations, reflections and translations:
    a. Angles are taken to angles of the same measure.
    b. Lines are taken to lines, and line segments to line segments of the same length
    c. Parallel lines are taken to parallel lines.
- 8.G.2 Understand that a two-dimensional figure is congruent to another if the second can be obtained from the first by a sequence of rotations, reflections, and translations; given two congruent figures, describe a sequence that exhibits the congruence between them.
- 8. G. 3 Describe the effect of dilations, translations, rotations, and reflections on two-dimensional figures using coordinates.
- 8.G .4 Understand that a two-dimensional figure is similar to another if the second can be obtained from the first by a sequence of rotations, reflections, translations, and dilations; given two similar two-dimensional figures, describe a sequence that exhibits the similarity between them.
- 8. G. 5 Use informal arguments to establish facts about the angle sum and exterior angle of triangles, about the angles created when parallel lines are cut by a transversal, and the angle-angle criterion for similarity of triangles. *For example, arrange three copies of the same triangle so that the sum of the three angles appears to form a line, and give an argument in terms of transversals why this is so.*

| **Understandings** | **Essential Questions** | |
|---|---|---|
| **Overarching Understanding** | **Overarching** | **Topical** |
| Students apply their understanding of the effect of geometric transformation(s) on a figure or shape.<br>Students describe how two figures or shapes are congruent or similar.<br>Students create or identify a sequence of transformations that lead to congruent or similar figures.<br>Students analyze the relationship between angles measures (triangle sum; parallel lines cut by a transversal; impact of a geometric transformation). | How are the (angles), (lengths), or (figures) changing? How are they staying the same?<br>How is ___ related to____?<br>What happens when an object is dilated?<br>How could an object be transformed to enlarge or reduce its size?<br>How can you determine the distance between two points in a coordinate plane? | How does what we measure affect how we measure?<br><br>How can space be defined through numbers/ measurement? |
| **Related Misconceptions** | | |

| | | |
|---|---|---|
| Students confuse the rules for transforming two-dimensional figures because they rely too heavily on rules as opposed<br>to understanding what happens to figures as they translate, rotate, reflect, and dilate. It is important to have students describe the effects of each of the transformations on two-dimensional figures through the coordinates but also the visual transformations that result.<br><br>Students have difficulty differentiating between congruency and similarity. Assume any combination of three angles will form a congruence condition.<br>Not recognize congruent figures because of different orientations.<br>Confuse terms such as clockwise and counter-clockwise. Think the line of reflection must be vertical or horizontal (e.g. across the y- axis or x-axis. Not realize that rotations are not always the origin, but can be about any point. | | |

| **Knowledge**<br>Students will know… | **Skills**<br>Students will be able to… |
|---|---|
| Congruence and similarity using physical models, transparencies or geometry software<br>Dilations , Translations , rotations and reflections on two dimensional figures using coordinates | Describe the effect of dilations, translations, rotations and reflections on two-dimensional figures using coordinates<br>Relationship between angle measures<br>Identify sequence of transformations |

## Assessment Evidence (Stage 2)

## Performance Task Description

| | |
|---|---|
| • **Goal**<br>• **Role**<br>• **Audience**<br>• **Situation**<br>• **Product/Performance**<br>• **Standards** | Resources/Tools:<br>8.G Reflecting a rectangle over a diagonal<br>8.G, G-GPE, G-SRT, G-CO Is this a rectangle?<br>8.G Partitioning a hexagon<br>8.G Same Size, Same Shape?<br>8.G, G-CO Origami Silver Rectangle<br>8.G Congruent Segments<br>8.G Congruent Rectangles<br>8.G Congruent Triangles<br>8.G Triangle congruence with coordinates<br>8.G Cutting a rectangle into two congruent triangles<br>8.G Circle Sandwich<br>8.G Reflecting reflections<br>8.G Triangle congruence with coordinates<br>8.G Point Reflection<br>8.G.A.3 Effects of Dilations on Length, Area, and Angles<br>8.G Find the Angle<br>8.G Find the Missing Angle<br>8.G Tile Patterns II: hexagons<br>8.G Tile Patterns I: octagons and squares<br>8.G A Triangle's Interior Angles<br>8.G Congruence of Alternate Interior Angles via Rotations<br>8.G.A.5 Street Intersections<br>8.G Rigid motions and congruent angles |
| **Other Evidence** | |
| | |
| **Learning Plan (Stage 3)** | |

- **Where are your students headed? Where have they been? How will you make sure the students know where they are going?**
- **How will you hook students at the beginning of the unit?**
- **What events will help students experience and explore the big idea and questions in the unit? How will you equip them with needed skills and knowledge?**
- **How will you cause students to reflect and rethink? How will you guide them in rehearsing, revising, and refining their work?**
- **How will you help students to exhibit and self-evaluate their growing skills, knowledge, and understanding throughout the unit?**
- **How will you tailor and otherwise personalize the learning plan to optimize the engagement and effectiveness of ALL students, without compromising the goals of the unit?**
- **How will you organize and sequence the learning activities to optimize the engagement and achievement of ALL students?**

## Standard: Grade 8.G.1

## Instructional Strategies:

A major focus in Grade 8 is to use knowledge of angles and distance to analyze two- and three- dimensional figures and space in order to solve problems. This cluster interweaves the relationships of symmetry, transformations, and angle relationships to form understandings of similarity and congruence.
Inductive and deductive reasoning are utilized as students forge into the world of proofs. Informal arguments are justifications based on known facts and logical
reasoning.
Students should be able to appropriately label figures, angles, lines, line segments, congruent parts, and images (primes or double primes). Students are expected to use logical thinking, expressed in words using correct terminology. They are NOT expected to use theorems, axioms, postulates or a formal format of proof as in two-column proofs.
Transformational geometry is about the effects of rigid motions, rotations, reflections and translations on figures. Initial work should be presented in such a way that students understand the concept of each type of transformation and the effects that each transformation has on an object before working within the coordinate system. For example, when reflecting over a line, each vertex is the same distance from the line as its corresponding vertex. This is easier to visualize when not using regular figures. Time should be allowed for students to cut out and trace the figures for each step in a series of transformations. Discussion should include the description of the relationship between the original figure and its image(s) in regards to their corresponding parts (length of sides and measure of angles) and the description of the movement, including the attributes of transformations (line of symmetry, distance to be moved, center of rotation, angle of rotation and the amount of dilation). The case of distance – preserving transformation leads to the idea of congruence.
It is these distance-preserving transformations that lead to the idea of congruence.

Work in the coordinate plane should involve the movement of various polygons by addition, subtraction and multiplied changes of the coordinates. For example, add 3 to $x$, subtract 4 from $y$, combinations of changes to $x$ and $y$, multiply coordinates by 2 then by ½.
Students should observe and discuss such questions as

| **Title of Unit** | **Function to Model Relationships between Quantities** | **Grade Level** | | |
|---|---|---|---|---|
| **Curriculum Area** | Mathematics | **Time Frame** | 4-5 weeks | |
| **Developed By** | Mathematics | | | |

## Identify Desired Results (Stage 1)

## Content Standards

**8.F.1**. Understand that a function is a rule that assigns to each input exactly one output. The graph of a function is the set of ordered pairs consisting of an input and the corresponding output.
**8.F.2** Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). For example, given a linear function represented by a table of values and a linear function represented by an algebraic expression, determine which function has the greater rate of change.
**8.F.3** Interpret the equation y = mx + b as defining a linear function, whose graph is a straight line; give examples of functions that are not linear. For example, the function A = s2 giving the area of a square as a function of its side length is not linear because its graph contains the points (1,1), (2,4) and (3,9), which are not on a straight line.
**8.G.6** Explain a proof of the Pythagorean Theorem and its converse.
**8.G.7** Apply the Pythagorean Theorem to determine unknown side lengths in right triangles in real world and mathematical problems in two and three dimensions.
**8.G.8** Apply the Pythagorean Theorem to find the distance between two points in a coordinate system

<table>
<tr><th>Understandings</th><th colspan="2">Essential Questions</th></tr>
<tr><th>Overarching Understanding</th><th>Overarching</th><th>Topical</th></tr>
<tr><td>Students understand that a function is a relationship with a unique output for each input.<br>Students develop their ability to make connections between multiple representations of functions and interpret the features of functions in terms of real world contexts.<br>Understand and apply the Pythagorean Theorem .<br>Students extend their understanding of the number system by investigating the relationship between the sides of a right triangle</td><td rowspan="3">How would you determine that a relationship is a function?<br>What are some characteristics of a (linear) (nonlinear) function?<br>How would you interpret the features (e.g. rate of change, initial value, increasing/ decreasing)of a function, in a real world context?</td><td rowspan="3">How is thinking algebraically different from thinking arithmetically?<br>• How do I use algebraic expressions to analyze or solve problems?<br>• How do the properties contribute to algebraic understanding?<br>•What is meant by equality?</td></tr>
<tr><th>Related Misconceptions</th></tr>
<tr><td>Some students will mistakenly think of a straight line as horizontal or vertical only.<br>Students may mistakenly believe that a slope of zero is the same as “no slope” and then confuse a horizontal line (slope of zero) with a vertical line (undefined slope).<br>Confuse the meaning of “domain” and “range” of a function. Some students will mix up x- and y-axes on the coordinate plane, or mix up the ordered pairs. When emphasizing that the first value is plotted on the horizontal axes (usually x, with positive to the right) and the second is the vertical axis (usually called y, with positive up), point out that this is merely a convention: It could have been otherwise, but it is very useful for people to agree on a standard customary practice<br>Student errors with the Pythagorean theorem may involve mistaking one of the legs as the hypotenuse, multiplying the legs and the hypotenuse by 2 as opposed to squaring them, or using the theorem to find missing sides for a triangle that is not right.</td></tr>
<tr><td>Knowledge<br>Students will know…</td><td colspan="2">Skills<br>Students will be able to…</td></tr>
</table>

| | |
|---|---|
| To define, evaluate and compare functions.<br>To use functions to model relationships between quantities.<br>Investigate patterns of association in bivariate data.<br>Understand and apply Pythagorean Theorem | Students grasp the concept of a function as a rule that assigns to each input exactly one output.<br>They understand that functions describe situations where one quantity determines another.<br>Students will apply real world problem using Pythagorean Theorem<br>Students approximate irrational numbers using their understanding of square and cube roots.<br>Students extend their understanding of the number system by investigating the relationship between the sides of a right triangle. |

## Assessment Evidence (Stage 2)

## Performance Task Description

| | |
|---|---|
| • **Goal**<br>• **Role**<br>• **Audience**<br>• **Situation**<br>• **Product/ Performance**<br>• **Standards** | 8.G Applying the Pythagorean Theorem in a mathematical context<br>8.G Bird and Dog Race<br>8.G, G-GPE, G-SRT, G-CO Is this a rectangle?<br>8.G A rectangle in the coordinate plane<br>8.G.B Sizing up Squares<br>8.G Converse of the Pythagorean Theorem<br>Maryland Geometry Lessons<br>Video Tutorial with Graph Paper (Pythagorean Theorem with 3-4-5 Triangle<br>8.G Glasses<br>8.G Spiderbox<br>8.G.7 Running on the Football Field<br>8.G Two Triangles' Area<br>8.G Area of a Trapezoid<br>8.G, G-SRT Points from Directions<br>8.G Areas of Geometric Shapes with the Same Perimeter 8.G Circle Sandwich<br>8.G Finding isosceles triangles<br>8.G Finding the distance between points<br>F-IF The Customers<br>8.F Foxes and Rabbits<br>8.F US Garbage, Version 1<br>6.EE,NS,RP; 8.EE,F Pennies to heaven<br>8.F Function Rules<br>8.F Introducing Functions<br>8.F Battery Charging |

## Other Evidence

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|---|
| Pythagorean Lessons: CCGPS Frameworks Geometric Applications of Exponents<br>For detailed information see Learning Progressions Functions:<br>http://commoncoretools.me/wp-content/uploads/2013/07/ccss_progression_functions_2013_07_02.pdf<br>Also see engageNY Modules: https://www.engageny.org/resource/grade-8-mathematics |
| **Learning Plan (Stage 3)** |

- **Where are your students headed? Where have they been? How will you make sure the students know where they are going?**
- **How will you hook students at the beginning of the unit?**
- **What events will help students experience and explore the big idea and questions in the unit? How will you equip them with needed skills and knowledge?**
- **How will you cause students to reflect and rethink? How will you guide them in rehearsing, revising, and refining their work?**
- **How will you help students to exhibit and self-evaluate their growing skills, knowledge, and understanding throughout the unit?**
- **How will you tailor and otherwise personalize the learning plan to optimize the engagement and effectiveness of ALL students, without compromising the goals of the unit?**
- **How will you organize and sequence the learning activities to optimize the engagement and achievement of ALL students?**

## Standard: Grade 8.G.6

## Instructional Strategies:

Previous understanding of triangles, such as the sum of two side measures is greater than the third side measure, angles sum, and area of squares, is furthered by the introduction of unique qualities of right triangles. Students should be given the opportunity to explore right triangles to determine the relationships between the measures of the legs and the measure of the hypotenuse. Experiences should involve using grid paper to draw right triangles from given measures
and representing and computing the areas of the squares on each side. Data should be recorded in a chart such as the one below, allowing for students to conjecture about the relationship among the areas within each triangle

| Triangle | Measure of Leg 1 | Measure of Leg 2 | Area of Square on Leg 1 | Area of Square on Leg 1 | Area of square on Hypotenuse |
|---|---|---|---|---|---|
| 1 | | | | | |

Students should then test out their conjectures, then explain and discuss their findings. Finally, the Pythagorean Theorem should be introduced and explained as the **pattern** they have explored. Time should be spent analyzing several proofs of the Pythagorean Theorem to develop a beginning sense of the process of deductive reasoning, these significance of a theorem, and the purpose of a proof. Students should be able to justify a simple proof of the Pythagorean Theorem or its converse.
Previously, students have discovered that not every combination of side lengths will create a triangle. Now they need situations that explore using the Pythagorean Theorem to test whether or not side lengths represent right triangles.
(Recording could include Side length *a*, Side length *b*, Sum of *a2* + *b2*, *c2*, *a2* + *b2* = c2, Right triangle? Through these opportunities, students should realize that there are Pythagorean (triangular) triples such as (3, 4, 5), (5, 12, 13), (7, 24, 25), (9, 40, 41) that always create right triangles, and that their multiples also form right triangles. Students should see how similar triangles can be used to find additional triples. Students should be able to explain why a triangle is or is not a right triangle using the Pythagorean Theorem.
The Pythagorean Theorem should be applied to finding the lengths of segments

| | | | |
|---|---|---|---|
| **Title of Unit** | Proportional Relationships and Linear Equations Involving Bivariate Data and Solution of Simultaneous Equations | **Grade Level** | $8^{th}$ Grade Level 2 Saturday |
| **Curriculum Area** | Mathematics | **Time Frame** | 4-5 weeks |
| **Developed By** | Munira Jamali | | |
| **Identify Desired Results (Stage 1)** | | | |
| **Content Standards** | | | |

- 8.F.4 Construct a function to model a linear relationship between two quantities. Determine the rate of change and initial value of the function from a description of a relationship or from two (x, y) values, including reading these from a table or from a graph. Interpret the rate of change and initial value of a linear function in terms of the situation it models, and in terms of its graph or a table of values.

- 8.F.5 Describe qualitatively the functional relationship between two quantities by analyzing a graph (e.g., where the function is increasing or decreasing, linear or nonlinear). Sketch a graph that exhibits the qualitative features of a function that has been described verbally

- 8.EE.5 Graph proportional relationships, interpreting the unit rate as the slope of the graph. Compare two different proportional relationships represented in different ways. For example, compare a distance-time graph to a distance-time equation to determine which of two moving objects has greater speed. 8.EE.6 Use similar triangles to explain why the slope m is the same between any two distinct points on a non-vertical line in the coordinate plane; derive the equation y = mx for a line through the origin and the equation y = mx + b for a line intercepting the vertical axis at b.

- 8.EE.7 Solve linear equations in one variable. a. Give examples of linear equations in one variable with one solution, infinitely many solutions, or no solutions. Show which of these possibilities is the case by successively transforming the given equation into simpler forms, until an equivalent equation of the form x = a, a = a, or a = b results (where a and b are different numbers). b. Solve linear equations with rational number coefficients, including equations whose solutions require expanding expressions using the distributive property and collecting like terms.

- 8.EE.8 Analyze and solve pairs of simultaneous linear equations.
  a. Understand that solutions to a system of two linear equations in two variables correspond to points of intersection of their graphs, because points of intersection satisfy both equations simultaneously.

| **Understandings** | **Essential Questions** | |
|---|---|---|
| **Overarching Understanding** | **Overarching** | **Topical** |
| Students compare proportional relationships using a variety of representations of these relationships (graph, table, symbols).<br>Students understand and represent slope as a unit rate, and apply their knowledge of right triangles to represent slope.<br>Students relate the slope with its concept as a rate and its visual representation as a set of right triangle that are similar for each line.<br>Students interpret slope and intercept using real world applications (e.g. bivariate data).<br>Students create equivalent equations to solve for an unknown.<br>Students employ graphical, tabular and symbolic representations to express linearity and determine the number of solutions.<br>Students interpret a linear equation in a real world application by deriving the equation. | How can I determine, when analyzing the motion of two objects, which object has the greater speed?<br>What is the meaning of the slope and intercept of a line, in the context of the situation?<br>How may I use similar triangles to show that the slope is the same, given two distinct sets of points on a graph?<br>How will I explain how I know that a pair of linear equations has one solution, no solutions, or infinitely many solutions? | How is thinking algebraically different from thinking arithmetically?<br>How do I use algebraic expressions to analyze or solve problems?<br>How do the properties contribute to algebraic understanding?<br>What is meant by equality |

| Related Misconceptions | many solutions.<br>Is the slope between any two points on the same line the same? Explain your reasoning.<br>How can I create an equation with given information from a table, graph, or problem situation?<br>How can mathematics be used to provide models that helps us interpret data and make predictions? | |
|---|---|---|
| Students think that only the letters x and y can be used for variables. Students think that you always need a variable = a constant as a solution. The variable is always on the left side of the equation. Equations are not always in the slope intercept form, $y=mx+b$. Students confuse one-variable and two-variable<br>equations | | |

| **Knowledge**<br>Students will know… | **Skills**<br>Students will be able to… |
|---|---|
| The connections between proportional relationships, lines and linear equations.<br>To investigate patterns of association in bivariate data.<br>To analyze and solve linear equations and pairs of simultaneous linear equations.<br>Define, evaluate and compare functions.<br>Use functions to model relationships between quantities | To construct a function to model a linear relationship.<br>Identify (from a graph, table, y= mx+b, etc.) and interpret the rate of change and initial value of a linear function in terms of the situation.<br>Solve linear equations in one variable with rational number coefficients.<br>Categorize linear equations in one variable as having one, none, or infinitely many solutions |

## Assessment Evidence (Stage 2)

## Performance Task Description

| | |
|---|---|
| • **Goal**<br>• **Role**<br>• **Audience**<br>• **Situation**<br>• **Product/Performance**<br>• **Standards** | **Resources/Tools:**<br>8-F Modeling with a Linear Function<br>8.F Heart Rate Monitoring<br>8.G Downhill<br>8.F Video Streaming<br>8.F High School Graduation<br>8.F Chicken and Steak, Variation 1<br>8.F Baseball Cards<br>8.F Chicken and Steak, Variation 2<br>8.F Distance across the channel<br>8.F Delivering the Mail, Assessment Variation<br>"Is it Fair?", Georgia Department of Education<br>8.F Tides<br>8.F Distance<br>8.F Bike Race<br>8.F Riding by the Library<br>8.EE Find the Change<br>8.EE Equations of Lines<br>8.EE DVD Profits, Variation 1<br>8.EE Proportional relationships, lines, and linear equations<br>8.EE Stuffing Envelopes<br>8.EE Folding a Square into Thirds<br>8.EE Coffee by the Pound<br>8.EE Peaches and Plums<br>8.EE Who Has the Best Job?<br>8.EE Comparing Speeds in Graphs and Equations<br>8.EE Sore Throats, Variation 2<br>8.EE Stuffing Envelopes<br>8.EE Two Lines<br>8.EE The Sign of Solutions<br>8.EE Coupon versus discount<br>8.EE Solving Equations<br>8.EE Sammy's Chipmunk and Squirrel Observations<br>8.EE How Many Solutions?<br>8.EE Fixing the Furnace<br>8.EE Cell Phone Plans<br>8.EE Kimi and Jordan<br>8.EE Folding a Square into Thirds<br>8.EE The Intersection of Two Lines<br>8.EE Quinoa Pasta 1<br>8.EE Summer Swimming |

## Other Evidence

"Cara's Candles and DVD's", Georgia Department of Education.

## Learning Plan (Stage 3)

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|---|---|
| • **Where are your students headed? Where have they been? How will you make sure the students know where they are going?**<br>• **How will you hook students at the beginning of the unit?**<br>• **What events will help students experience and explore the big idea and questions in the unit? How will you equip them with needed skills and knowledge?**<br>• **How will you cause students to reflect and rethink? How will you guide them in rehearsing, revising, and refining their work?**<br>• **How will you help students to exhibit and self-evaluate their growing skills, knowledge, and understanding throughout the unit?**<br>• **How will you tailor and otherwise personalize the learning plan to optimize the engagement and effectiveness of ALL students, without compromising the goals of the unit?**<br>• **How will you organize and sequence the learning activities to optimize the engagement and achievement of ALL students?** | **Grade 8.F.4**<br>**Instructional Strategies:**<br>In Grade 8, students focus on linear equations and functions. Nonlinear functions are used for comparison.<br>Students will need many opportunities and examples to figure out the meaning of $y = mx + b$.<br>What does $m$ mean? What does $b$ mean? They should be able to "see" $m$ and $b$ in graphs, tables, and formulas or equations, and they need to be able to interpret those values in contexts. For example, if a function is used to model the height of a stack of $n$ paper cups, then the rate of change, $m$, which is the slope of the graph, is the height of the "lip" of the cup: the amount each cup sticks above the lower cup in the stack. The "initial value" in this case is not valid in the context because 0 cups would not have a height, and yet a height of 0 would not fit the equation. Nonetheless,<br>the value of $b$ can be interpreted in the context as the height of the "base" of the cup: the height of the whole cup minus its lip.<br>Use graphing calculators and web resources to explore linear and non-linear functions. Provide context as much as possible to build understanding of slope and y-intercept in a graph, especially for those patterns that do not start with an initial value of 0.<br>Give students opportunities to gather their own data or graphs in contexts they understand. Students need to measure, collect data, graph data, and look for patterns, then generalize and symbolically represent the patterns. They also need opportunities to draw graphs (qualitatively, based upon experience) representing real-life situations with which they are familiar. Probe student thinking by asking them to determine which input values make sense in the problem situations<br>given.<br>Provide students with a function in symbolic form and ask them to create a problem situation in words to match the function. Given a graph, have students create a scenario that would fit the graph. Ask students to sort a set of "cards" to match a graphs, tables, equations, and problem situations. Have students explain their reasoning to each other.<br>From a variety of representations of functions, students should be able to classify and describe the function as linear or non-linear, increasing or decreasing. Provide opportunities for students to share their ideas with other students and create their own examples for their classmates.<br>Use the slope of the graph and similar triangle arguments to call attention to not just the change in $x$ or $y$, but also to therate of change, which is a ratio of the two.<br>Emphasize key vocabulary. Students should be able to explain what key words mean: e.g., model, interpret, initial value, functional relationship, qualitative, |

| Title of Unit | Statistics and Geometry | Grade Level | 8th Grade Level 2 Saturday |
|---|---|---|---|
| Curriculum Area | Mathematics | Time Frame | 3-4 weeks |
| Developed By | Munira Jamali | | |

## Identify Desired Results (Stage 1)

## Content Standards

- **8.SP.1** Construct and interpret scatter plots for bivariate measurement data to investigate patterns of association between two quantities. Describe patterns such as clustering, outliers, positive or negative association, linear association, and nonlinear association.
- **8.SP.2** Know that straight lines are widely used to model relationships between two quantitative variables. For scatter plots that suggest a linear association, informally fit a straight line, and informally assess the model fit by judging the closeness of the data points to the line.
- **8.SP.4** Understand that patterns of association can also be seen in bivariate categorical data by displaying frequencies and relative frequencies in a two-way table. Construct and interpret a two-way table summarizing data on two categorical variables collected from the same subjects. Use relative frequencies calculated for rows or columns to describe possible association between the two variables. For example, collect data from students in your class on whether or not they have a curfew on school nights and whether or not they have assigned chores at home. Is there evidence that those who have a curfew also tend to have chores?
- **8.G.9** Know the formulas for the volumes of cones, cylinders, and spheres and use them to solve real-world and mathematical problems.

| **Understandings** | **Essential Questions** | |
|---|---|---|
| **Overarching Understanding** | **Overarching** | **Topical** |
| Students understand that a function is a relationship with a unique output for each input.<br>Students develop their ability to make connections between multiple representations of functions and interpret the features of functions in terms of real world contexts.<br>Students are able to construct a function to model a linear relationship.<br>Students identify (from a graph, table, y= mx+b, etc.) and interpret the rate of change and initial value of a linear function in terms of the situation. | How would you determine that a relationship is a function? What are some characteristics of a (linear) (nonlinear) function? How would you interpret the features (e.g. rate of change, initial value, increasing/ decreasing) of a function, in a real world context?<br>How would you determine, depict, and describe "patterns of association" between two quantities, in bivariate data? | • Geometry and spatial sense offer ways to interpret and reflect on our physical environment.<br>• Analyzing geometric relationships develops reasoning and justification skills |
| **Related Misconceptions** | | |

| | | |
|---|---|---|
| Students may believe Bivariate data is only displayed in scatter plots. Grade 8.SP.4 in this cluster provides the<br>opportunity to display bivariate, categorical data in a table.<br>In general, students think there is only one correct answer in mathematics. Students may mistakenly think their lines of best fit for the same set of data will be exactly the same. Because students are informally drawing lines of best fit, the<br>lines will vary slightly. To obtain the exact line of best fit, students would use technology to find the line of regression | | |

| **Knowledge**<br>Students will know… | **Skills**<br>Students will be able to… |
|---|---|
| Define, evaluate and compare functions.<br>Investigate patterns of association in bivariate data.<br>MP 1, 4, 5, 6, and 7<br>Solve real-world and mathematical problems involving volume of cylinders | • Determine the Volume<br>• Analyze data<br>• Solve real life problem |

## Assessment Evidence (Stage 2)

## Performance Task Description

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| • **Goal**<br>• **Role**<br>• **Audience**<br>• **Situation**<br>• **Product/Performance**<br>• **Standards** | 8.G Comparing Snow Cones<br>8.G Glasses<br>8.G Flower Vases<br>8.G Shipping Rolled Oats<br>8.SP Birds' Eggs<br>8-SP.1 Texting and Grades I<br>8.SP.1 Hand span and height<br>8.SP Animal Brains<br>8.SP Birds' Eggs<br>8.SP Animal Brains<br>8.SP Laptop Battery Charge |

## Other Evidence

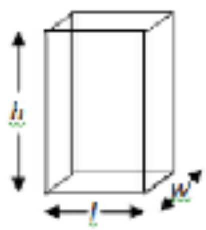
# Learning Plan (Stage 3)

- **Where are your students headed? Where have they been? How will you make sure the students know where they are going?**
- **How will you hook students at the beginning of the unit?**
- **What events will help students experience and explore the big idea and questions in the unit? How will you equip them with needed skills and knowledge?**
- **How will you cause students to reflect and rethink? How will you guide them in rehearsing, revising, and refining their work?**
- **How will you help students to exhibit and self-evaluate their growing skills, knowledge, and understanding throughout the unit?**
- **How will you tailor and otherwise personalize the learning plan to optimize the engagement and effectiveness of ALL students, without compromising the goals of the unit?**
- **How will you organize and sequence the learning activities to optimize the engagement and achievement of ALL students?**

## Standard: Grade 8.G.9

## Instructional Strategies:

Begin by recalling the formula, and its meaning, for the volume of a right rectangular prism: $V = l \times w \times h$. Then ask students to consider how this might be used to make a conjecture about the volume formula for a cylinder.





Most students can be readily led to the understanding that the volume of a right rectangular prism can be thought of as the area of a base times the height, and so because the area of the base of a cylinder is $\pi r2$ the volume of a cylinder is $V_c = \pi r^2 h$.

To motivate the formula for the volume of a cone, use cylinders and cones with the same base and height. Fill the cone with rice or water and pour into the cylinder. Students will discover/experience that 3 cones full are needed to fill the cylinder. This non-mathematical derivation of the formula for the volume of a cone will help most students

remember the formula.

In a drawing of a cone inside a cylinder, students might see that that the triangular cross-section of a cone is 1/2the rectangular cross-section of the cylinder. Ask them to reason why the volume (three dimensions) turns out to be *less* than ½ the volume of the cylinder. It turns out to be 1/3.

For the volume of a sphere, it may help to have students visualize a hemisphere "inside" a cylinder with the same height and base.‖ The radius of the circular base of the cylinder is also the radius of the sphere and the hemisphere. The area of the base of the cylinder and the area of the section created by the division of the sphere into a