Title of Unit	Relationships between Quantities and Reasoning with Equations	**Grade Level**	Grade 8^{th} Level 3 Saturday
Curriculum Area	Mathematics	**Time Frame**	3-4 weeks
Developed By	Munira Jamali		

Identify Desired Results (Stage 1)

Content Standards

Algebra - Seeing Structure in Expressions
A.SSE.1 Interpret expressions that represent a quantity in terms of its context.
★ a. Interpret parts of an expression, such as terms, factors, and coefficients.
b. Interpret complicated expressions by viewing one or more of their parts as single entity. For example, interpret P(1+r)n as the product of P and a factor not depending on P.
Algebra - Reasoning with Equations and Inequalities
A.REI.1 Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method.
A.REI.3 Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.
A.REI.3.1 Solve one-variable equations and inequalities involving absolute value, graphing the solutions and interpreting them in context.
Numbers - Quantities
N.Q.1 Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays.
N.Q.2 Define appropriate quantities for the purpose of descriptive modeling.
N.Q.3 Choose a level of accuracy appropriate to limitations on measurement when reporting quantities.
Algebra - Creating Equations
A.CED.1 Create equations and inequalities in one variable and use them to solve problems. Include equations arising from linear and quadratic functions, and simple rational and exponential functions.
A.CED.2 Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.
A.CED.3 Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or non-viable options in a modeling context. For example, represent inequalities describing nutritional and cost constraints on combinations of different foods. A.CED.4 Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations. For example, rearrange Ohm’s law V = IR to highlight resistance R.

Understandings	**Essential Questions**	
Overarching Understanding	**Overarching**	**Topical**

Understand that numbers in real world applications often have units attached to them, and they are considered quantities. Understand the structure of algebraic expressions and polynomials. · Understand general linear equations ($y=mx+b$, $m\neq 0$) and their graphs and extend this to work with absolute value equations, linear inequalities, and systems of linear equations. · Use properties of equality and order of operation to solve an equation by using inverse operations. · Solve equations and inequalities give all the values of a variable that make the equation/inequality true. · The values that define inequalities are graphically represented by either: a set of linear values or the areas represented above or below the linear values. **Related Misconceptions**	What are the "pieces" of an algebraic expression? What do they represent in the context of the real-world situation? What do the parts of an expression tell us in a real-world context? How would you describe the difference between an expression and an equation? How do the properties of equality and order of operations extend to support the solving of an equation? · Why is	**When and how is mathematics used in solving real world problems** *How are equations and inequalities used to solve real world problems?* **What characteristics of problems would determine how to model the situation and develop a problem solving strategy?** *What characteristics of problems would help to distinguish whether the situation could be modeled by a linear or an exponential model?* *When is it advantageous to represent relationships between quantities symbolically? numerically?*

Common Misconceptions: N.RN.1-2 Students sometimes misunderstand the meaning of exponential operations, the way powers and roots relate to one another, and the order in which they should be performed. Attention to the base is very important. Consider examples: $(-81^{3/4})$ and $(-81)^{3/4}$. The position of a negative sign of a term with a rational exponent can mean that the rational exponent should be either applied first to the base, 81, and then the opposite of the result is taken, $(-81^{3/4})$, or the rational exponent should be applied to a negative term $(-81)^{3/4}$. The answer of 4 -81 will be not real if the denominator of the exponent is even. If the root is odd, the answer will be a negative number. Students should be able to make use of estimation when incorrectly using multiplication instead of exponentiation. Students may believe that the fractional exponent in the expression $36^{\frac{1}{3}}$ means the same as a factor of $1/3$ in multiplication expression, $36 \bullet 1/3$ and multiple the base by the exponent. **Common Misconceptions: N.Q.1-3** Students may not realize the importance of the units' conversions in conjunction with the computation when solving problems involving measurements. Students often have difficulty understanding how ratios expressed in different units can be equal to one. For example, $\frac{5280 ft}{1 mile}$ is simply one, and it is permissible to multiply by that ratio. Students need to make sure to put the quantities in the numerator or denominator so that the terms can cancel appropriately. Since today's calculating devices often display 8 to 10 decimal places, students frequently express answers to a much greater degree of precision than the required. **Common Misconceptions**: **A.SSE.1-2** Students may believe that the use of algebraic expressions is merely the abstract manipulation of symbols. Use of real- world context examples to demonstrate the meaning of the parts of	it important to be able to solve linear equations and inequalities in one variable? How do you graphically represent the solutions to a linear equation? How do you graphically represent the values that define linear inequalities	*graphically?* **Why is it necessary to follow set rules/procedures/ properties when manipulating numeric or algebraic expressions?** *How can, the structure of expressions/equations/ inequalities, units of measure used, and mathematical properties help determine a solution strategy?*

Knowledge Students will know…	**Skills** Students will be able to…
Interpret the structure of expressions Understand solving equations as a process of reasoning and explain the reasoning. Solve equations and inequalities in one variable. Reason quantitatively and use units to solve problems.	To analyze and explain the process of solving an equation. Develop fluency writing, interpreting, and translating between various forms of linear equations and inequalities, and using them to solve problems Master the solution of linear equations and apply related solution techniques and the laws of exponents to the creation and solution of simple exponential equations. All of this work is grounded on understanding quantities and on relationships between them.
Assessment Evidence (Stage 2)	
Performance Task Description	

• **Goal** • **Role** • **Audience** • **Situation** • **Product/ Performance** • **Standards**	**Solving Equations in One Variable: (8.EE)** http://map.mathshell.org/materials/lessons.php?taskid=442 Goals Students are able to: Solve linear equations in one variable with rational number coefficients. • Collect like terms. • Expand expressions using the distributive property. • Categorize linear equations in one variable as having one, none, or infinitely many solutions. It also aims to encourage discussion on some common misconceptions about algebra. **8.EE: Analyze and solve linear equations and pairs of simultaneous linear equations.** **Sorting Equations and Identities** http://map.mathshell.org/materials/lessons.php?taskid=218&subpage=concept Goals • Recognize the differences between equations and identities. • Substitute numbers into algebraic statements in order to test their validity in special cases. • Resist common errors when manipulating expressions such as $2(x - 3) = 2x - 3$; $(x + 3)^2 = x^2 + 3^2$. • Carry out correct algebraic manipulations. It also aims to encourage discussion on some common misconceptions about algebra A-REI: Solve equations and inequalities in one variable
Other Evidence	
Manipulating Polynomials: http://map.mathshell.org/materials/lessons.php?taskid=437&subpage=concept Defining Regions of Inequalities: http://map.mathshell.org/materials/lessons.php?taskid=219&subpage=concept Interpreting Algebraic Expressions: http://map.mathshell.org/materials/lessons.php?taskid=219&subpage=concept	
Learning Plan (Stage 3)	

- **Where are your students headed? Where have they been? How will you make sure the students know where they are going?**
- **How will you hook students at the beginning of the unit?**
- **What events will help students experience and explore the big idea and questions in the unit? How will you equip them with needed skills and knowledge?**
- **How will you cause students to reflect and rethink? How will you guide them in rehearsing, revising, and refining their work?**
- **How will you help students to exhibit and self-evaluate their growing skills, knowledge, and understanding throughout the unit?**
- **How will you tailor and otherwise personalize the learning plan to optimize the engagement and effectiveness of ALL students, without compromising the goals of the unit?**
- **How will you organize and sequence the learning activities to optimize the engagement and achievement of ALL students?**

Instructional Strategies: N.Q.1-3
In real-world situations, answers are usually represented by numbers associated with units. Units involve measurement and often require a conversion. Measurement involves both precision and accuracy. Estimation and approximation often precede more exact computations. Students need to develop sound mathematical reasoning skills and forms of argument to make reasonable judgments about their solutions. They should be able to decide whether a problem calls for an estimate, for an approximation, or for an exact answer. To accomplish this goal, teachers should provide students with a broad range of contextual problems that offer opportunities for performing operations with quantities involving units. These problems should be connected to science, engineering, economics, finance, medicine, etc. Some contextual problems may require an understanding of derived measurements and capability in unit analysis. Keeping track of derived units during computations and making reasonable estimates and rational conclusions about accuracy and the precision of the answers help in the problem-solving process. For example, while driving in the United Kingdom (UK), a U.S. tourist puts 60 liters of gasoline in his car. The gasoline cost is £1.28 per liter The exchange rate is £ 0.62978 for each $1.00. The price for a gallon of a gasoline in the United States is $3.05. The driver wants to compare the costs for the same amount and the same type of gasoline when he/she pays in UK pounds. Making reasonable estimates should be encouraged prior to solving this problem. Since the current exchange rate has inflated the UK pound at almost twice the U.S. dollar, the driver will pay more for less gasoline. By dividing $3.05 by 3.79L (the number of liters in one gallon), students can see that 80.47 cents per liter of gasoline in US is less expensive than £1.28 or $ 2.03 per liter of the same type of gasoline in the UK when paid in U.S. dollars. The cost of 60 liters of gasoline in UK is In order to compute the cost of the same quantity of gasoline in the United

$$\pounds 30.41 \ \left(\frac{US\$3.05}{1\,gal} \times \frac{1\,gal}{3.79\,L} \times 60L \times \frac{UK\,\pounds 0.62978}{US\,\$1.00} = UK\,\pounds 30.41\right).$$

States in UK
The computation shows that the gasoline is less expensive in the United States and how an analysis can be helpful in keeping track of unit conversations. Students should be able to correctly identify the degree of precision of the answers which should not be far greater than the actual accuracy of the measurements. Graphical representations serve as visual models for understanding phenomena that take place in our daily surroundings. The use of different kinds of graphical representations along with their units, labels and titles demonstrate the level of students' understanding and foster the ability to reason, prove, self-check, examine relationships and establish the validity of arguments. Students need to be able to identify misleading graphs by choosing correct units and scales to create a correct representation of a situation or to make a correct conclusion from it.

Explanations and Examples: N.Q.1
Solution:
The student must choose the following four rates of quantities (order does not matter):

20 L, 1,000 mL, 0.82 g and 1 Kg

Title of Unit	Linear Relationships	**Grade Level**	8th grade Level 3 Supplemental
Curriculum Area	Mathematics	**Time Frame**	3-4 Weeks
Developed By	Munira Jamali		
Identify Desired Results (Stage 1)			
Content Standards			

Algebra - Reasoning with Equations and Inequalities
A.REI.10. Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line).
A.REI.11. Explain why the x-coordinates of the points where the graphs of the equations y=f(x) and y=g(x) intersect are the solutions of the equation f(x) = g(x) find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where f(x) and/ or g(x) are linear, polynomial, rational, absolute value, exponential, and logarithmic functions. ★
A.REI.12. Graph the solutions to a linear inequality in two variables as a half-plane (excluding the boundary in the case of a strict inequality), and graph the solution set to a system of linear inequalities in two variables as the intersection of the corresponding half-planes.
A-REI.5. Prove that, given a system of two equations in two variables, replacing one equation by the sum of that equation and a multiple of the other produces a system with the same solutions.
A-REI.6. Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables
A.CED.3 Represent constraints by equations or inequalities, and by the systems of equations and/or inequalities, and interpret solutions as viable or non-viable options in a modeling content. For example, represent inequalities describing nutritional and cost constraints on combinations of different foods.
Functions - Interpreting Functions
F.IF.1. Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If f is a function and x is an element of its domain, then f(x) denotes the output of f corresponding to the input x. The graph of f is the graph of the equation y = f(x).
F.IF.2. Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.
F.IF.3. Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers. For example, the Fibonacci sequence is defined recursively by f(0) = f(1) = 1, f(n + 1) = f(n) + f(n – 1) for n ≥ 1.
F.IF.7. Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. a. Graph linear and quadratic functions and show intercepts, maxima, and minima. b. Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions. ★
F.IF.9. Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). For example, given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum

Understandings	Essential Questions	
Overarching Understanding	**Overarching**	**Topical**

Apply rules so that polynomials form a system analogous to integers. Write in equivalent forms that represent both linear and exponential functions and construct functions to describe the situation and to find solutions Apply rules that builds a function that models a relationship between two quantities Represent equations and inequalities in one variable in various ways and use them to extend the properties of exponents to rational exponents Understand the relationship between quantities of two systems of equations and the methods to solve two system of linear equations Model with linear and exponential functions. Systems of equations compare at least two different functions Write in equivalent forms that represent both linear and exponential functions and construct functions to describe the situation and to find solutions Apply rules that builds a function that models a relationship between two quantities Represent equations and inequalities in one variable in various ways and use them to extend the properties of exponents to rational exponents Understand the relationship between quantities of two systems of equations and the methods to solve two system of linear equations	How will students identify the different parts of a two system equation and explain their meaning within the context of the problem? 2. What is the importance of identifying the structure of functions and using different ways to represent them? 3. Why is it important to identify and extend the properties of exponents to rational exponents? 4. When do students decide the best method to solve an inequality? 5. How do you know which method to use in solving a system of equations? 6. Why is it important to analyze functions using different representations? 7 How do I analyze algebraic equations/ inequalities to solve problems? 8. What must students understand in order to create equations that describe numbers or relationships?	What does the slope of a line indicate about the line? What information does the equation of a line give you? How are equations and graphs related?
Related Misconceptions		

Common Misconceptions: A.REI.1-2 Students may believe that solving an equation such as 3x + 1 = 7 involves "only removing the 1," failing to realize that the equation 1 = 1 is being subtracted to produce the next step. Additionally, students may believe that all solutions to radical and rational equations are viable, without recognizing that there are times when extraneous solutions are generated and have to be eliminated. **Common Misconceptions: A.REI.10-12** Students may believe that the graph of a function is simply a line or curve "connecting the dots," without recognizing that the graph represents all solutions to the equation. Students may also believe that graphing linear and other functions is an isolated skill, not realizing that multiple graphs can be drawn to solve equations involving those functions. Additionally, students may believe that two-variable inequalities have no application in the real world. Teachers can consider business related problems (e.g., linear programming applications) to engage students in discussions of how the inequalities are derived and how the feasible set includes all the points that satisfy the conditions stated in the inequalities. **Common Misconceptions: F.IF.1-3** Students may believe that all relationships having an input and an output are functions, and therefore, misuse the function terminology. Students may also believe that the notation f(x) means to multiply some value f times another value x. The notation alone can be confusing and needs careful development. For example, f(2) means the output value of the function f when the input value is 2. **Common Misconceptions: F.IF.7-9** Students may believe that each family of functions (e.g., quadratic, square root, etc.) is independent of the others, so they may not recognize commonalities among all functions and their graphs. Students may also believe that skills such as factoring a trinomial or completing the square are isolated within a unit on polynomials, and that they will come to understand the usefulness of these skills in the context of examining characteristics of functions. Additionally, student may believe that the process of rewriting equations into various forms is simply an algebra symbol manipulation exercise, rather than serving a purpose of allowing different features of the function to be exhibited. **Common Misconceptions: F.LE.1-4** Students may believe that all functions have a first common difference and need to explore to realize that, for example, a quadratic function will have equal second common differences in a table. Students may also believe that the end behavior of all functions depends on the situation and not the fact that exponential function values will eventually get larger than those of any other polynomial functions.	9. How do students know the most efficient ways to build a function that models a relationship between two quantities? 10. Why is it important to understand solving a system of linear and exponential relationships in two variables algebraically and graphically? 11. Is there functional relationship in non-linear and ambiguous data? 12. What is the difference in linear and exponential functions and how is that represented graphically? 13. What real-life situations would need exponential or linear function functions to describe them? 14. What is the relationship of a recursive function on the table and graph that represents it? 15. How might an arithmetic sequence be connected to a linear function? 16. How might a geometric sequence be connected to an exponential function??	

Knowledge Students will know…	**Skills** Students will be able to…
Extend the properties of exponents to rational exponents. Build a function that models a relationship between two quantities. Build new functions from existing functions Interpret functions that arise in applications in terms of a context. Solve systems of equations. Represent and solve equations and inequalities Graphically	Develop the concepts of domain and range in function notation. They move beyond viewing functions as processes that take inputs and yield outputs and start viewing functions as objects in their own right. Explore many examples of functions, including sequences; they interpret functions given graphically, numerically, symbolically, and verbally, translate between representations, and understand the limitations of various representations. Work with functions given by graphs and tables, keeping in mind that, depending upon the context, these representations are likely to be approximate and incomplete. Their work includes functions that can be described or approximated by formulas as well as those that cannot. When functions describe relationships between quantities arising from a context, students reason with the units in which those quantities are measured. Explore systems of equations and inequalities, and they find and interpret their solutions. Students build on and informally extend their understanding of integer exponents to consider exponential functions. They compare and contrast linear and exponential functions, distinguishing between additive and multiplicative change. Interpret arithmetic sequences as linear functions and geometric sequences as exponential functions
Assessment Evidence (Stage 2)	
Performance Task Description	

• **Goal** • **Role** • **Audience** • **Situation** • **Product/ Performance** • **Standards**	• Building and Solving Equations 2: A-REI http://map.mathshell.org/materials/lessons.php?taskid=554#task554 Goals Solving equations where the unknown appears once or more than once Solving equations in more than one way Standard A-REI: Understand solving equations as a process of reasoning and explain the reasoning. Solve equations and inequalities in one variable http://www.ccsstoolbox.com/parcc/PARCCPrototype_main.html Celllular Growth Goals Students write geometric sequences both recursively and with an explicit formula and use the two representations to model situations. Students consider the use of one or more representations to solve a real-world problem and choose the type of sequence to represent a situation. Rabbit Growth Goals Student will use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another.
Other Evidence	
College Board http://achieve.lausd.net/cms/lib08/CA01000043/Centricity/domain/244/alignment%20doc/Algebra%201%20Textbook%20Aligment%20-%20SpringBoard.pdf Engage New York http://www.engageny.org/sites/default/files/resource/attac hments/algebra-i-m1-copy-ready-materials.pdf Illustrative Mathematics A Sum of Functions – F. BF. 1a http://www.illustrativemathematics.org/illustrations/230	
Learning Plan (Stage 3)	

- **Where are your students headed? Where have they been? How will you make sure the students know where they are going?**
- **How will you hook students at the beginning of the unit?**
- **What events will help students experience and explore the big idea and questions in the unit? How will you equip them with needed skills and knowledge?**
- **How will you cause students to reflect and rethink? How will you guide them in rehearsing, revising, and refining their work?**
- **How will you help students to exhibit and self-evaluate their growing skills, knowledge, and understanding throughout the unit?**
- **How will you tailor and otherwise personalize the learning plan to optimize the engagement and effectiveness of ALL students, without compromising the goals of the unit?**
- **How will you organize and sequence the learning activities to optimize the engagement and achievement of ALL students?**

Instructional Strategies: A.REI.10-12

Beginning with simple, real-world examples, help students to recognize a graph as a set of solutions to an equation. For example, if the equation $y = 6x + 5$ represents the amount of money paid to a babysitter (i.e., \$5 for gas to drive to the job and \$6/hour to do the work), then every point on the line represents an amount of money paid, given the amount of time worked.

Explore visual ways to solve an equation such as $2x + 3 = x - 7$ by graphing the functions $y = 2x + 3$ and $y = x - 7$. Students should recognize that the intersection point of the lines is at (-10, -17). They should be able to verbalize that the intersection point means that when $x = -10$ is substituted into both sides of the equation, each side simplifies to a value of -17. Therefore, -10 is the solution to the equation. This same approach can be used whether the functions in the original equation are linear, nonlinear or both.

Using technology, have students graph a function and use the trace function to move the cursor along the curve. Discuss the meaning of the ordered pairs that appear at the bottom of the calculator, emphasizing that every point on the curve represents a solution to the equation.

Begin with simple linear equations and how to solve them using the graphs and tables on a graphing calculator. Then, advance students to nonlinear situations so they can see that even complex equations that might involve quadratics, absolute value, or rational functions can be solved fairly easily using this same strategy. While a standard graphing calculator does not simply solve an equation for the user, it can be used as a tool to approximate solutions.

Use the table function on a graphing calculator to solve equations. For example, to solve the equation $x^2 = x + 12$, students can examine the equations $y = x^2$ *and* $y = x + 12$ and determine that they intersect when $x = 4$ and when $x = -3$ by examining the table to find where the *y*-values are the same.

Investigate real-world examples of two-dimensional inequalities. For example, students might explore what the graph would look like for money earned when a person earns *at least* \$6 per hour. (The graph for a person earning *exactly* \$6/hour would be a linear function, while the graph for a person earning at least \$6/hour would be a half-plane including the line and all points above it .

Explanations and Examples: A.REI.10

In Algebra 1 for A.REI.10 focus on linear and exponential equations and be able to adapt and apply that learning to other types of equations in future courses.

Students can explain and verify that every point (x, y) on the graph of an equation represents values x and y that make the equation true

Examples:

Which of the following points is on the circle with equation $(x-1)^2 + (y+2)^2 = 5$?

(a) (1, -2) **(b)** (2, 2) **(c)** (3, -1) **(d)** (3, 4)

Graph the equation and determine which of the following points are on the graph of $y = 3^x + 1$.

(a) (2,7) **(b)** (-1, 4/3) **(c)** (2, 10) **(d)** (0, 1)

Title of Unit	Function and Modeling	Grade Level	Level 3
Curriculum Area	Mathematics	Time Frame	3-4 weeks
Developed By	Munira Jamali		
Identify Desired Results (Stage 1)			
Content Standards			

A-SSE.1 Interpret expressions that represent a quantity in terms of its context.★ a. Interpret parts of an expression, such as terms, factors, and coefficients. b. Interpret complicated expressions by viewing one or more of their parts as a single entity. For example, interpret P(1+r)n as the product of P and a factor not depending on P.
A-SSE.2 Use the structure of an expression to identify ways to rewrite it. For example, see $x^4 - y^4$ as $(x^2)^2 - (y^2)^2$, thus recognizing it as a difference of squares that can be factored as $(x^2 - y^2)(x^2 + y^2)$.
A-SSE.3 Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.★
a. Factor a quadratic expression to reveal the zeros of the function it defines.
b. Complete the square in a quadratic expression to reveal the maximum or minimum value of the function it defines.
c. Use the properties of exponents to transform expressions for exponential functions. For example the expression 1.15t can be rewritten as $(1.15^{1/2})^{12t} \approx 1.012^{12t}$ to reveal the approximate equivalent monthly interest rate if the annual rate is 15%.
A-APR.1 Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials.
A-CED.1 Create equations and inequalities in one variable and use them to solve problems. Include equations arising from linear and quadratic functions, and simple rational and exponential functions.
A-CED.2 Create equations in two or more variables to represent relationships graph equations on coordinate axes with labels and scales.
A-CED.4 Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations. For example, rearrange Ohm's law V = IR to highlight resistance R.
A-REI.4 Solve quadratic equations in one variable. a. Use the method of completing the square to transform any quadratic equation in x into an equation of the form (x – p) 2 = q that has the same solutions. Derive the quadratic formula from this form. b. Solve quadratic equations by inspection (e.g., for $x^2 = 49$), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as a ± bi for real numbers a and b

Understandings	Essential Questions

Overarching Understanding	Overarching	Topical
Represent a quantity in terms of an expression, such as terms, factors, and coefficients by viewing one or more of their parts as a single entity. · Write in equivalent forms to find solutions that reveal and explain properties of quadratic expressions from completing the square, factoring, and using properties of exponents. · Apply rules so that polynomials form a system analogous to integers. · Represent equations and inequalities in one variable in various ways and use them to solve problems. • Understand the relationship between quantities of two or more variables through graphing on a coordinate plane system. • Transform quadratic equations using the method of completing the square to derive a solution. • Recognize the various methods to solve quadratic equations stemming from an initial form as appropriate: taking the square root, completing the square, using the quadratic formula, and factoring. • Identify when the quadratic formula gives complex solutions. • Solve systems of linear equations in two variables algebraically and graphically.	1. How will students identify the different parts of an expression and explain their meaning within the context of the problem? 2. What is the importance of identifying the structure of an expression and ways to rewrite it? 3. Why is it important to solve and produce equivalent forms of an expression? 4. When is factoring the best method to solve a quadratic equation? 5. When is completing the square useful to reveal the maximum or minimum value of the function it defines? 6. How do students know which method to use in solving quadratic equations? 7. Why is it important to know the operations of integers to understand the properties of	Why do we use equations to solve problems? Why do you perform operations on ***both*** sides of an equation? How is thinking algebraically different from thinking arithmetically? How do the properties contribute to algebraic understanding?
Related Misconceptions		

properties of polynomials?

Common Misconceptions: A.SSE.1-2

Students may believe that the use of algebraic expressions is merely the abstract manipulation of symbols. Use of real- world context examples to demonstrate the meaning of the parts of algebraic expressions is needed to counter this misconception.

Students may also believe that an expression cannot be factored because it does not fit into a form they recognize. They need help with reorganizing the terms until structures become evident.
Students will often combine terms that are not like terms. For example,
$2 + 3x = 5x$ or $3x + 2y = 5xy$.
Students sometimes forget the coefficient of 1 when adding like terms. For example, $x + 2x + 3x = 5x$ rather than $6x$.
Students will change the degree of the variable when adding/subtracting like terms. For example, $2x + 3x = 5x^2$ rather than $5x$.
Students will forget to distribute to all terms when multiplying. For example, $6(2x + 1) = 12x + 1$ rather than $12x + 6$.
Students may not follow the Order of Operations when simplifying expressions. For example, $4x^2$ when $x = 3$ may be incorrectly evaluated as $4\cdot3^2 = 12^2 = 144$, rather than $4\cdot9 = 36$. Another common mistake occurs when the distributive property should be used prior to adding/subtracting. For example, $2 + 3(x - 1)$ incorrectly becomes $5(x - 1) = 5x - 5$ instead of $2 + 3(x - 1) = 2 + 3x - 3 = 3x - 1$. Students fail to use the property of exponents correctly when using the distributive property. For example, $3x(2x - 1) = 6x - 3x = 3x$ instead of simplifying as $3x(2x - 1) = 6x$ 2 $- 3x$. Students fail to understand the structure of expressions. For example, they will write $4x$ when $x = 3$ is 43 instead of $4x = 4\cdot x$ so when $x = 3$, $4x = 4\cdot3 = 12$. In addition, students commonly misevaluate $-3^2 = 9$ rather than $-3^2 = -9$. Students routinely see -3 2 as the same as $(-3)^2 = 9$. A method that may clear up the misconception is to have students rewrite as $-x^2 = -1\cdot x^2$ so they know to apply the exponent before the multiplication of –1.
Students frequently attempt to "solve" expressions. Many students add "= 0" to an expression they are asked to simplify. Students need to understand the difference between an equation and an expression. Students commonly confuse the properties of exponents, specifically the product of powers property with the power of a power property. For example, students will often simplify $(x^2)^3 = x^5$ instead of x^6. Students will incorrectly translate expressions that contain a difference of terms. For example, 8 less than 5 times a number is often incorrectly translated as $8 - 5n$ rather than $5n - 8$.

A.APR.1

Some students will apply the distributive property inappropriately. Emphasize that it is the *distributive property of multiplication over addition*. For

8. How can students analyze algebraic equations/inequalities to solve problems?
9. What must students understand in order to create equations that describe numbers or relationships?
10. How do students know which is the most effective way to solve a quadratic equation ?
11. Why is it important to understand solving a system of linear and quadratic equations in two variables algebraically and graphically?
12. How are the methods of solving a quadratic equation related?
13. How do students know when the roots of a quadratic equation are real or complex?
14. Why are the methods of solving quadratic equations not learned in isolation?

Knowledge Students will know…	**Skills** Students will be able to…
To interpret the structure of expressions To write expressions in equivalent forms to solve problems. Perform arithmetic operations on polynomials. Create equations that describe numbers or relationships. Solve equations and inequalities in one variable. Solve systems of equations.	Build on their knowledge from Unit 2, where they extended the laws of exponents to rational exponents. Apply this new understanding of numbers and strengthen their ability to see structure in and create quadratic and exponential expressions. Create and solve equations, inequalities, and systems of equations involving quadratic expressions and determine the values of the function it defines. Understand that polynomials form a system analogous to the integers, they choose and produce equivalent forms of an expression

Assessment Evidence (Stage 2)	
Performance Task Description	
• **Goal** • **Role** • **Audience** • **Situation** • **Product/Performance** • **Standards**	**Solving Linear Equations in Two Variables** http://map.mathshell.org/materials/download.php?fileid=694 This lesson unit is intended to help you assess how well students are able to formulate and solve problems using algebra and in particular, to identify and help students who have the following difficulties: • Solving a problem using two linear equations with two variables. • Interpreting the meaning of algebraic expressions. A-SSE: Interpret the structure of expressions. A-REI: Solve systems of equations. **Solving Linear Equations in Two Variables** http://map.mathshell.org/download.php?fileid=1730 This lesson unit is intended to help you assess how well students are able to formulate and solve problems using algebra and in particular, to identify and help students who have the following difficulties: • Solving a problem using two linear equations with two variables. • Interpreting the meaning of algebraic expressions. A-SSE: Interpret the structure of expressions. A-REI: Solve systems of equations. Sorting Equations and Identities http://map.mathshell.org/download.php?fileid=1720 This lesson unit is intended to help you assess how well students are able to: • Recognize the differences between equations and identities. • Substitute numbers into algebraic statements in order to test their validity in special cases. • Resist common errors when manipulating expressions such as $2(x - 3) = 2x - 3$; $(x + 3)^2 = x^2 + 3^2$. • Carry out correct algebraic manipulations.

Other Evidence

Enage New York
Algebra I Module 4: Polynomial and Quadratic Expressions, Equations, and Function
llustrative Mathematics
http://www.illustrativemathematics.org/standard s/hs
http://www.wiki-teacher.com/ Math Resources – algebra
Influenza Epidemic – F.IF.4
http://www.illustrativemathematics.org/illustrations/637

• Warming and Cooling – F.IF.4:
http://www.illustrativemathematics.org/illustrations/639

• How is the weather – F.IF.4:
http://www.illustrativemathematics.org/illustrations/649

• Logistic Growth Model, Explicit Version – F.IF.4
http://www.illustrativemathematics.org/illustrations/804

• The Canoe Trip, Variation 1 – F.IF.4-5
http://www.illustrativemathematics.org/illustrations/386

Learning Plan (Stage 3)

- **Where are your students headed? Where have they been? How will you make sure the students know where they are going?**
- **How will you hook students at the beginning of the unit?**
- **What events will help students experience and explore the big idea and questions in the unit? How will you equip them with needed skills and knowledge?**
- **How will you cause students to reflect and rethink? How will you guide them in rehearsing, revising, and refining their work?**
- **How will you help students to exhibit and self-evaluate their growing skills, knowledge, and understanding throughout the unit?**
- **How will you tailor and otherwise personalize the learning plan to optimize the engagement and effectiveness of ALL students, without compromising the goals of the unit?**
- **How will you organize and sequence the learning activities to optimize the engagement and achievement of ALL students?**

Instructional Strategies: A.SSE.3

It is important to balance conceptual understanding and procedural fluency in work with equivalent expressions. For example, development of skill in factoring and completing the square goes hand-in-hand with understanding what different forms of a quadratic expression reveal.

This cluster focuses on linking expressions and functions, i.e., creating connections between multiple representations of functional relations – the dependence between a quadratic expression and a graph of the quadratic function it defines, and the dependence between different symbolic representations of exponential functions. Teachers need to foster the idea that changing the forms of expressions, such as factoring or completing the square, or transforming expressions from one exponential form to another, are not independent algorithms that are learned for the sake of symbol manipulations. They are processes that are guided by goals (e.g., investigating properties of families of functions and solving contextual problems).

Factoring methods that are typically introduced in elementary algebra and the method of completing the square reveals attributes of the graphs of quadratic functions, represented by quadratic equations.

The solutions of quadratic equations solved by factoring are the x – intercepts of the parabola or zeros of quadratic functions.

A pair of coordinates (h, k) from the general form $f(x) = a(x – h)^2 + k$ represents the vertex of the parabola, where h represents a horizontal shift and k represents a vertical shift of the parabola $y = x^2$ from its original position at the origin.

A vertex (h, k) is the minimum point of the graph of the quadratic function if $a > 0$ and is the maximum point of the graph of the quadratic function if $a < 0$.

Understanding an algorithm of completing the square provides a solid foundation for deriving a quadratic formula.

Translating among different forms of expressions, equations and graphs helps students to understand some key connections among arithmetic, algebra and geometry. The reverse thinking technique (a process that allows working backwards from the answer to the starting point) can be very effective. Have students derive information about a function's equation, represented in standard, factored or general form, by investigating its graph.

Offer multiple real-world examples of exponential functions. For instance, to illustrate an exponential decay, students need to recognize that in the equation for an automobile cost $C(t) = 20{,}000(0.75)^t$, the base is 0.75 and between 0 and 1 and the value of $20,000 represents the initial cost of an automobile that depreciates 25% per year over the course of t years.

Similarly, to illustrate exponential growth, in the

Title of Unit	Descriptive Statistics	8^{th} grade	Level 3
Curriculum Area	Mathematics	Time Frame	2-3 weeks
Developed By	Munira Jamali		

Identify Desired Results (Stage 1)

Content Standards

Statistics and Probability - Interpreting Categorical and Quantitative Data
S.ID.1 Represent data with plots on the real number line (dot plots, histograms, and box plots)
S.ID.2 Use statistics appropriate to the shape of the data distribution to compare center (median, mean) and spread (interquartile range, standard deviation) of two or more different data sets.
S.ID.3 Interpret differences in shape, center, and spread in the context of the data sets, accounting for possible effects of extreme data points (outliers)
S.ID.4 Use the mean and standard deviation of a data set to fit it to a normal distribution and to estimate population percentages
Statistics and Probability - Interpreting Categorical and Quantitative Data
S.ID.5 Summarize categorical data for two categories in two-way frequency tables. Interpret relative frequencies in the context of the data (including joint, marginal, and conditional relative frequencies). Recognize possible associations and trends in the data.
S.ID.6 Represent data on two quantitative variables on a scatter plot, and describe how the variables are related. a. Fit a function to the data; use functions fitted to data to solve problems in the context of the data. Use given functions or choose a function suggested by the context. Emphasize linear and exponential models. b. Informally assess the fit of a function by plotting and analyzing residuals. c. Fit a linear function for a scatter plot that suggests a linear association.
S.ID.7 Interpret the slope (rate of change) and the intercept (constant term) of a linear model in the context of the data.
S.ID.8 Compute (using technology) and interpret the correlation coefficient of a linear fit.
S.ID.9 Distinguish between correlation and causation.

Understandings	Essential Questions	
Overarching Understanding	Overarching	Topical

<table>
<tr>
<td>· A linear function can be used to model the relationship between two numerical variables.
· The strength of a relationship and appropriateness of the model used can be determined by analyzing residuals.
· A statistical relationship, such as correlation coefficient, is not necessarily the same as a cause and-effect relationship.
· The correlation coefficient will be understood and the focus will be on the computation and interpretation of the correlation coefficient as a measure of how well the data fit the relationship.
· A deeper look at bivariate data can be taken to describe categorical associations and how to fit models to quantitative data</td>
<td rowspan="3">How would you analyze bivariate data using your knowledge of proportions?
How would you describe categorical variables?
How would you use your knowledge of functions to fit models to quantitative data? How would you interpret the parameters of a linear model in the context of data that it represents? How can you compute correlation coefficients using technology and interpret the value of the coefficient?
How do analysis of bivariate data and knowledge of proportions intersect with each other?</td>
<td rowspan="3">How do you interpret data on a graph?
How do you distinguish between a population and a sample?
How do you design an experiment?
How do you conduct a probability experiment?
What is conditional probability? How do you determine if 2 events are mutually exclusive?</td>
</tr>
<tr>
<td>Related Misconceptions</td>
</tr>
<tr>
<td>Common Misconceptions: S.CP.6-7
Students may believe: That the probability of A or B is always the sum of the two events individually. That the probability of A and B is the product of the two events individually, not realizing that one of the probabilities may be conditional.
S.ID.1-4
Students may believe:
That a bar graph and a histogram are the same. A bar graph is appropriate when the horizontal axis has categories and the vertical axis is labeled by either frequency (e.g., book titles on the horizontal and number of students who like the respective books on the vertical) or measurement of some numerical variable (e.g., days of the week on the horizontal and median length of root growth of radish seeds on the vertical). A histogram has units of measurement of a numerical variable on the horizontal (e.g., ages with intervals of equal length).
That the lengths of the intervals of a boxplot (min,Q1), (Q1,Q2), (Q2,Q3), (Q3,max) are related to the number of subjects in each interval. Students should understand that each interval theoretically contains one-fourth of the total number of subjects. Sketching an accompanying histogram and constructing a live boxplot may help in alleviating this misconception.</td>
</tr>
</table>

<table>
<tr><th>Knowledge
Students will know…</th><th>Skills
Students will be able to…</th></tr>
<tr><td>To display numerical data and summarize it using measures of center and variability. By the end of middle school they were creating scatterplots and recognizing linear trends in data.
To build upon that prior experience, providing students with more formal means of assessing how a model fits data.
To use regression techniques to describe approximately linear relationships between quantities.
To use graphical representations and knowledge of the context to make judgments about the appropriateness of linear models. With linear models, they look at residuals to analyze the goodness of fit.</td><td>Summarize, represent, and interpret data on a single count or measurement variable.
In grades 6 – 8, students describe center and spread in a data distribution. Here they choose a summary statistic appropriate to the characteristics of the data distribution, such as the shape of the distribution or the existence of extreme data points

Summarize, represent, and interpret data on two categorical and quantitative variables.
Students take a more sophisticated look at using a linear function to model the relationship between two numerical variables. In addition to fitting a line to data, students assess how well the model fits by analyzing residuals. S.ID.6b should be focused on linear models, but may be used to preview quadratic functions in Unit 4 of this course

Interpret linear models.
Build on students' work with linear relationships in eighth grade and introduce the correlation coefficient. The focus here is on the computation and interpretation of the correlation coefficient as a measure of how well the data fit the relationship. The important distinction between a statistical
relationship and a cause-and-effect relationship arises in S.ID.9</td></tr>
<tr><th colspan="2">Assessment Evidence (Stage 2)</th></tr>
<tr><th colspan="2">Performance Task Description</th></tr>
<tr><td>Goal
Role
Audience
Situation
Product/ Performance
Standards</td><td>Representing Data with Frequency Graphs
http://map.mathshell.org/materials/download.php?fileid=1230
Goals
Students are able to
Are able to use frequency graphs to identify a range of measures and make sense of this data in a real-world context.
• Understand that a large number of data points allow a frequency graph to be approximated by a continuous distribution.
Standard
S-ID: Summarize, represent, and interpret data on a single count or measurement variable.
Representing Data with Box Plots
http://map.mathshell.org/download.php?fileid=1782
Students are able
To interpret data using frequency graphs and box plots
To identify and help students who have difficulty figuring out the data points and spread of data from frequency graphs and box plots</td></tr>
</table>

Other Evidence

Interpreting Statistics: A Case of Muddying the Waters – S.ID 7-9
http://map.mathshell.org/materials/download.php?fileid=686
ILLUSTRIVE MATHEMATICS
• Speed Trap – S.ID.1, 2, 3:
http://www.illustrativemathematics.org/illustrations/1027
Coffee and Crime – S.ID.6-9:
http://www.illustrativemathematics.org/illustrations/1307
• Olympic Men's 100-meter dash – S.ID.6a, 7:
http://www.illustrativemathematics.org/illustrations/1554
• Used Subaru Foresters I – S.ID.6a:
http://www.illustrativemathematics.org/illustrations/941
• Texting and Grades II – S.ID.7
https://www.illustrativemathematics.org/content-standards/tasks/1028

Learning Plan (Stage 3)

Where are your students headed? Where have they been? How will you make sure the students know where they are going? **How will you hook students at the beginning of the unit?** **What events will help students experience and explore the big idea and questions in the unit? How will you equip them with needed skills and knowledge?** **How will you cause students to reflect and rethink? How will you guide them in rehearsing, revising, and refining their work?** **How will you help students to exhibit and self-evaluate their growing skills, knowledge, and understanding throughout the unit?** **How will you tailor and otherwise personalize the learning plan to optimize the engagement and effectiveness of ALL students, without compromising the goals of the unit?** **How will you organize and sequence the learning activities to optimize the engagement and achievement of ALL students?**	**Instructional Strategies: S.ID.1-4** Have students practice their understanding of the different types of graphs for categorical and numerical variables by constructing statistical posters. Note that a bar graph for categorical data may have frequency on the vertical (student's pizza preferences) or measurement on the vertical (radish root growth over time - days). Measures of center and spread for data sets without outliers are the mean and standard deviation, whereas median and interquartile range are better measures for data sets with outliers. Introduce the formula of standard deviation by reviewing the previously learned MAD (mean absolute deviation). The MAD is very intuitive and gives a solid foundation for developing the more complicated standard deviation measure. Informally observing the extent to which two boxplots or two dot plots overlap begins the discussion of drawing inferential conclusions. Don't shortcut this observation in comparing two data sets. As histograms for various data sets are drawn, common shapes appear. To characterize the shapes, curves are sketched through the midpoints of the tops of the histogram's rectangles. Of particular importance is a symmetric unimodal curve that has specific areas within one, two, and three standard deviations of its mean. It is called the Normal distribution and students need to be able to find areas (probabilities) for various events using tables or a graphing calculator. **Explanations and Examples: S.ID.1** A statistical process is a problem-solving process consisting of four steps: 2. formulating a statistical question that anticipates variability and can be answered by data 2. designing and implementing a plan that collects appropriate data. 3. analyzing the data by graphical and/or numerical methods. 4. interpreting the analysis in the context of the original question. Graph numerical data on a real number line using dot plots, histograms, and box plots. Analyze the strengths and weakness inherent in each type of plot by comparing different plots of the same data. Describe and give a simple interpretation of a graphical representation of data. **Examples:** The following data set shows the number of songs downloaded in one week by each student in Mrs. Jones class: 10, 20, 12, 14, 12, 27, 88, 2, 7, 30, 16, 16, 32, 25, 15, 4, 0, 15, 6. Choose and create a plot to represent the data. On the midterm math exam, students had the following scores: 95, 45, 37, 82, 90, 100, 91, 78, 67, 84, 85, 85, 82, 91, 93, 92, 76, 84, 100, 59, 92, 77, 68, and 88. What are the strengths and weaknesses of presenting this