Title of Unit	Linear Relationships	Grade Level	12th grade SAT prep
Curriculum Area	Mathematics	Time Frame	5 weeks
Developed By	Munira Jamali		
Identify Desired Results (Stage 1)			
Content Standards			

Algebra - Reasoning with Equations and Inequalities

A.REI.10. Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line).

A.REI.11. Explain why the x-coordinates of the points where the graphs of the equations y=f(x) and y=g(x) intersect are the solutions of the equation f(x) = g(x) find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where f(x) and/ or g(x) are linear, polynomial, rational, absolute value, exponential, and logarithmic functions. ★

A.REI.12. Graph the solutions to a linear inequality in two variables as a half-plane (excluding the boundary in the case of a strict inequality), and graph the solution set to a system of linear inequalities in two variables as the intersection of the corresponding half-planes.

A-REI.5. Prove that, given a system of two equations in two variables, replacing one equation by the sum of that equation and a multiple of the other produces a system with the same solutions.

A-REI.6. Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables

A.CED.3 Represent constraints by equations or inequalities, and by the systems of equations and/or inequalities, and interpret solutions as viable or non-viable options in a modeling content. For example, represent inequalities describing nutritional and cost constraints on combinations of different foods.

Functions - Interpreting Functions

F.IF.1. Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If f is a function and x is an element of its domain, then f(x) denotes the output of f corresponding to the input x. The graph of f is the graph of the equation y = f(x).

F.IF.2. Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.

F.IF.3. Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers. For example, the Fibonacci sequence is defined recursively by f(0) = f(1) = 1, f(n + 1) = f(n) + f(n – 1) for n ≥ 1.

F.IF.7. Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. a. Graph linear and quadratic functions and show intercepts, maxima, and minima. b. Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions. ★

F.IF.9. Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). For example, given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum

Understandings	Essential Questions	
Overarching Understanding	**Overarching**	**Topical**
Apply rules so that polynomials form a system analogous to integers. Write in equivalent forms that represent both linear and exponential functions and construct functions to describe the situation and to find solutions Apply rules that builds a function that models a relationship between two quantities Represent equations and inequalities in one variable in various ways and use them to extend the properties of exponents to rational exponents Understand the relationship between quantities of two systems of equations and the methods to solve two system of linear equations Model with linear and exponential functions. Systems of equations compare at least two different functions Write in equivalent forms that represent both linear and exponential functions and construct functions to describe the situation and to find solutions Apply rules that builds a function that models a relationship between two quantities Represent equations and inequalities in one variable in various ways and use them to extend the properties of exponents to rational exponents Understand the relationship between quantities of two systems of equations and the methods to solve two system of linear equations	How will students identify the different parts of a two system equation and explain their meaning within the context of the problem? 2. What is the importance of identifying the structure of functions and using different ways to represent them? 3. Why is it important to identify and extend the properties of exponents to rational exponents? 4. When do students decide the best method to solve an inequality? 5. How do you know which method to use in solving a system of equations? 6. Why is it important to analyze functions using different representations? 7 How do I analyze algebraic equations/ inequalities to solve problems? 8. What must students understand in order to create equations that	What does the slope of a line indicate about the line? What information does the equation of a line give you? How are equations and graphs related?
Related Misconceptions		

Common Misconceptions: A.REI.1-2 Students may believe that solving an equation such as 3x + 1 = 7 involves "only removing the 1," failing to realize that the equation 1 = 1 is being subtracted to produce the next step. Additionally, students may believe that all solutions to radical and rational equations are viable, without recognizing that there are times when extraneous solutions are generated and have to be eliminated. **Common Misconceptions: A.REI.10-12** Students may believe that the graph of a function is simply a line or curve "connecting the dots," without recognizing that the graph represents all solutions to the equation. Students may also believe that graphing linear and other functions is an isolated skill, not realizing that multiple graphs can be drawn to solve equations involving those functions. Additionally, students may believe that two-variable inequalities have no application in the real world. Teachers can consider business related problems (e.g., linear programming applications) to engage students in discussions of how the inequalities are derived and how the feasible set includes all the points that satisfy the conditions stated in the inequalities. **Common Misconceptions: F.IF.1-3** Students may believe that all relationships having an input and an output are functions, and therefore, misuse the function terminology. Students may also believe that the notation f(x) means to multiply some value f times another value x. The notation alone can be confusing and needs careful development. For example, f(2) means the output value of the function f when the input value is 2. **Common Misconceptions: F.IF.7-9** Students may believe that each family of functions (e.g., quadratic, square root, etc.) is independent of the others, so they may not recognize commonalities among all functions and their graphs. Students may also believe that skills such as factoring a trinomial or completing the square are isolated within a unit on polynomials, and that they will come to understand the usefulness of these skills in the context of examining characteristics of functions. Additionally, student may believe that the process of rewriting equations into various forms is simply an algebra symbol manipulation exercise, rather than serving a purpose of allowing different features of the function to be exhibited. **Common Misconceptions: F.LE.1-4** Students may believe that all functions have a first common difference and need to explore to realize that, for example, a quadratic function will have equal second common differences in a table. Students may also believe that the end behavior of all functions depends on the situation and not the fact that exponential function values will eventually get larger than those of any other polynomial functions	describe numbers or relationships? 9. How do students know the most efficient ways to build a function that models a relationship between two quantities? 10. Why is it important to understand solving a system of linear and exponential relationships in two variables algebraically and graphically? 11. Is there functional relationship in non-linear and ambiguous data? 12. What is the difference in linear and exponential functions and how is that represented graphically? 13. What real-life situations would need exponential or linear function functions to describe them? 14. What is the relationship of a recursive function on the table and graph that represents it? 15. How might an arithmetic sequence be connected to a linear function? 16. How might a geometric sequence be connected to an exponential function??		

Knowledge Students will know…	**Skills** Students will be able to…
Extend the properties of exponents to rational exponents. Build a function that models a relationship between two quantities. Build new functions from existing functions Interpret functions that arise in applications in terms of a context. Solve systems of equations. Represent and solve equations and inequalities Graphically	Develop the concepts of domain and range in function notation. They move beyond viewing functions as processes that take inputs and yield outputs and start viewing functions as objects in their own right. Explore many examples of functions, including sequences; they interpret functions given graphically, numerically, symbolically, and verbally, translate between representations, and understand the limitations of various representations. Work with functions given by graphs and tables, keeping in mind that, depending upon the context, these representations are likely to be approximate and incomplete. Their work includes functions that can be described or approximated by formulas as well as those that cannot. When functions describe relationships between quantities arising from a context, students reason with the units in which those quantities are measured. Explore systems of equations and inequalities, and they find and interpret their solutions. Students build on and informally extend their understanding of integer exponents to consider exponential functions. They compare and contrast linear and exponential functions, distinguishing between additive and multiplicative change. Interpret arithmetic sequences as linear functions and geometric sequences as exponential functions
Assessment Evidence (Stage 2)	
Performance Task Description	

• **Goal** • **Role** • **Audience** • **Situation** • **Product/ Performance** • **Standards**	**Linear and Exponential Functions** **Topic A** **Linear and Exponential Sequences** **Focus Standards:** F-IF.A.1 Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If f is a function and xx is an element of its domain, then f(x) denotes the output of $f$corresponding to the input xx. The graph of f is the graph of the equation y = (xx). F-IF.A.2 Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context. F-IF.A.3 Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers. For example, the Fibonacci sequence is defined recursively by (0) = (1) = 1, (n + 1) = (n) + f(n– 1) for nn ≥ 1. F-IF.B.6 Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.★ F-BF.A.1 Write a function that describes a relationship between two quantities.★ a. Determine an explicit expression, a recursive process, or steps for calculation from a context. F-LE.A.1 Distinguish between situations that can be modeled with linear functions and with exponential functions.★ a. Prove that linear functions grow by equal differences over equal intervals, and that exponential functions grow by equal factors over equal intervals. b. Recognize situations in which one quantity changes at a constant rate per unit interval relative to another. c. Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another. F-LE.A.2 Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).★ F-LE.A.3 Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically, or (more generally) as a polynomial function Instructional Days: 7 Lesson 1: Integer Sequences—Should You Believe in Patterns? (P)1 Lesson 2: Recursive Formulas for Sequences (P) Lesson 3: Arithmetic and Geometric Sequences (P) Lesson 4: Why Do Banks Pay YOU to Provide Their Services? (P) Lesson 5: The Power of Exponential Growth (S) Lesson 6: Exponential Growth —U.S. Population and World Population (M) Lesson 7: Exponential Decay (P) **Topic B Functions and Their Graphs** **Focus Standards:** F-IF.A.1 Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If f is a function and x is an element of its domain, then (xx) denotes the output of f corresponding to the input xx. The graph of ff is the graph of the equation yy = (x).

Other Evidence
SAT Released Tests 1-7
Learning Plan (Stage 3)

• Where are your students headed? Where have they been? How will you make sure the students know where they are going? • How will you hook students at the beginning of the unit? • What events will help students experience and explore the big idea and questions in the unit? How will you equip them with needed skills and knowledge? • How will you cause students to reflect and rethink? How will you guide them in rehearsing, revising, and refining their work? • How will you help students to exhibit and self-evaluate their growing skills, knowledge, and understanding throughout the unit? • How will you tailor and otherwise personalize the learning plan to optimize the engagement and effectiveness of ALL students, without compromising the goals of the unit? • How will you organize and sequence the learning activities to optimize the engagement and achievement of ALL students?	In Lesson 1 of Topic A, students challenge the idea that patterns can be defined by merely seeing the first few numbers of the pattern. They learn that a sequence is an ordered list of elements and that it is sometimes intuitive to number the elements in a sequence beginning with 0 rather than 1. In Lessons 2 and 3, students learn to define sequences explicitly and recursively and begin their study of arithmetic and geometric sequences that continues through Lessons 4-7 as students explore applications of geometric sequences. In the final lesson, students compare arithmetic and geometric sequences as they compare growth rates. Throughout this topic, students use the notation of functions without naming it as such—they come to understand $f(n)$ as a “formula for the nth term of a sequence,” expanding to use other letters such as $AA(nn)$ for Akelia’s sequence and $BB(n)$ for Ben’s sequence. Their use of this same notation for functions is developed in TopicB. In Lesson 8, students consider that the notation they have been using to write explicit formulas for sequences can be applied to situations where the inputs are not whole numbers. In Lessons 9 and 10, they revisit the notion of function that was introduced in Grade 8. They are now prepared to use function notation as they write functions, interpret statements about functions, and evaluate functions for inputs in their domains. They formalize their understanding of a function as a correspondence between two sets, X and Y, in which each element of X is matched (or assigned) to one and only one element of Y, and add the understanding that the set X is called the domain, and the set Y is called the range. Students study the graphs of functions in Lessons 11–14 of this topic. In Lesson 11, students learn the meaning of the graph of a function, f, as the set of all points $(x, (x))$ in the Cartesian plane, such that x is in the domain of f and $f(x)$ is the value assigned to xx by the correspondence of the function. Students use plain English to write the instructions needed to plot the graph of a function. The instructions are written in a way similar to writing computer “pseudocode”—before actually writing the computer programs. In Lesson 12, students learn that the graph of $= (x)$ is the set of all points (x, y) in the plane that satisfy the equation $y = (x)$ and conclude that it is the same as the graph of the function explored in Lesson 11. In Lesson 13, students use a graphic of the planned landing sequence Mars Curiosity Rover to create graphs of specific aspects of the landing sequence—altitude over time and velocity over time—and use the graphs to examine the meaning of increasing and decreasing functions. Finally, Lesson 14 capitalizes on students’ new knowledge of functions and their graphs to contrast linear and exponential functions and the growth rates which they model. Lesson 15 of this topic formalizes the study of piecewise functions that began in Module 1. The study of piecewise functions in this lesson includes step functions and the

<table>
<tr><td>Title of Unit</td><td>Function and Modeling</td><td>Grade Level</td><td>12th grade SAT Prep</td></tr>
<tr><td>Curriculum Area</td><td>Mathematics</td><td>Time Frame</td><td>3 weeks</td></tr>
<tr><td>Developed By</td><td colspan="3">Munira Jamali</td></tr>
<tr><td colspan="4">Identify Desired Results (Stage 1)</td></tr>
<tr><td colspan="4">Content Standards</td></tr>
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<table>
<tr><td colspan="3">A-SSE.1 Interpret expressions that represent a quantity in terms of its context.★ a. Interpret parts of an expression, such as terms, factors, and coefficients. b. Interpret complicated expressions by viewing one or more of their parts as a single entity. For example, interpret P(1+r)n as the product of P and a factor not depending on P.
A-SSE.2 Use the structure of an expression to identify ways to rewrite it. For example, see $x^4 - y^4$ as $(x^2)^2 - (y^2)^2$, thus recognizing it as a difference of squares that can be factored as $(x^2 - y^2)(x^2 + y^2)$.
A-SSE.3 Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.★
a. Factor a quadratic expression to reveal the zeros of the function it defines.
b. Complete the square in a quadratic expression to reveal the maximum or minimum value of the function it defines.
c. Use the properties of exponents to transform expressions for exponential functions. For example the expression 1.15t can be rewritten as $(1.15^{1/2})^{12t} \approx 1.012^{12t}$ to reveal the approximate equivalent monthly interest rate if the annual rate is 15%.
A-APR.1 Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials.
A-CED.1 Create equations and inequalities in one variable and use them to solve problems. Include equations arising from linear and quadratic functions, and simple rational and exponential functions.
A-CED.2 Create equations in two or more variables to represent relationships graph equations on coordinate axes with labels and scales.
A-CED.4 Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations. For example, rearrange Ohm's law V = IR to highlight resistance R.
A-REI.4 Solve quadratic equations in one variable. a. Use the method of completing the square to transform any quadratic equation in x into an equation of the form (x – p) 2 = q that has the same solutions. Derive the quadratic formula from this form. b. Solve quadratic equations by inspection (e.g., for $x^2 = 49$), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as a ± bi for real numbers a and b</td></tr>
<tr><td>Understandings</td><td colspan="2">Essential Questions</td></tr>
<tr><td>Overarching Understanding</td><td>Overarching</td><td>Topical</td></tr>
</table>

Represent a quantity in terms of an expression, such as terms, factors, and coefficients by viewing one or more of their parts as a single entity. • Write in equivalent forms to find solutions that reveal and explain properties of quadratic expressions from completing the square, factoring, and using properties of exponents. • Apply rules so that polynomials form a system analogous to integers. • Represent equations and inequalities in one variable in various ways and use them to solve problems. • Understand the relationship between quantities of two or more variables through graphing on a coordinate plane system. • Transform quadratic equations using the method of completing the square to derive a solution. • Recognize the various methods to solve quadratic equations stemming from an initial form as appropriate: taking the square root, completing the square, using the quadratic formula, and factoring. • Identify when the quadratic formula gives complex solutions. • Solve systems of linear equations in two variables algebraically and graphically	1. How will students identify the different parts of an expression and explain their meaning within the context of the problem? 2. What is the importance of identifying the structure of an expression and ways to rewrite it? 3. Why is it important to solve and produce equivalent forms of an expression? 4. When is factoring the best method to solve a quadratic equation? 5. When is completing the square useful to reveal the maximum or minimum value of the function it defines? 6. How do students know which method to use in solving quadratic equations? 7. Why is it important to know the operations of integers to understand the properties of polynomials? 8. How can students	Why do we use equations to solve problems? Why do you perform operations on ***both*** sides of an equation? How is thinking algebraically different from thinking arithmetically? How do the properties contribute to algebraic understanding?
Related Misconceptions		

Common Misconceptions: A.SSE.1-2
Students may believe that the use of algebraic expressions is merely the abstract manipulation of symbols. Use of real- world context examples to demonstrate the meaning of the parts of algebraic expressions is needed to counter this misconception.

Students may also believe that an expression cannot be factored because it does not fit into a form they recognize. They need help with reorganizing the terms until structures become evident.
Students will often combine terms that are not like terms. For example,
2 + 3x = 5x or 3x + 2y = 5xy.
Students sometimes forget the coefficient of 1 when adding like terms. For example, x + 2x + 3x = 5x rather than 6x.
Students will change the degree of the variable when adding/subtracting like terms. For example, 2x + 3x = $5x^2$ rather than 5x.
Students will forget to distribute to all terms when multiplying. For example, 6(2x + 1) = 12x + 1 rather than 12x + 6.
Students may not follow the Order of Operations when simplifying expressions. For example, $4x^2$ when x = 3 may be incorrectly evaluated as $4•3^2$ = 12^2 = 144, rather than 4•9 = 36. Another common mistake occurs when the distributive property should be used prior to adding/subtracting. For example, 2 + 3(x – 1) incorrectly becomes 5(x - 1) = 5x - 5 instead of 2 + 3(x – 1) = 2 + 3x – 3 = 3x – 1. Students fail to use the property of exponents correctly when using the distributive property. For example, 3x(2x – 1) = 6x – 3x = 3x instead of simplifying as 3x(2x – 1) = 6x 2 – 3x. Students fail to understand the structure of expressions. For example, they will write 4x when x = 3 is 43 instead of 4x = 4•x so when x = 3, 4x = 4•3 = 12. In addition, students commonly misevaluate $–3^2$ = 9 rather than -3^2 = –9. Students routinely see –3 2 as the same as $(–3)^2$ = 9. A method that may clear up the misconception is to have students rewrite as $-x^2$ = $–1•x^2$ so they know to apply the exponent before the multiplication of –1.
Students frequently attempt to "solve" expressions. Many students add "= 0" to an expression they are asked to simplify. Students need to understand the difference between an equation and an expression.
Students commonly confuse the properties of exponents, specifically the product of powers property with the power of a power property. For example, students will often simplify $(x^2)^3 = x^5$ instead of x^6. Students will incorrectly translate expressions that contain a difference of terms. For example, 8 less than 5 times a number is often incorrectly translated as 8 – 5n rather than 5n – 8.

8. How can students analyze algebraic equations/inequalities to solve problems?
9. What must students understand in order to create equations that describe numbers or relationships?
10. How do students know which is the most effective way to solve a quadratic equation ?
11. Why is it important to understand solving a system of linear and quadratic equations in two variables algebraically and graphically?
12. How are the methods of solving a quadratic equation related?
13. How do students know when the roots of a quadratic equation are real or complex?
14. Why are the methods of solving quadratic equations not learned in isolation?

Knowledge Students will know…	**Skills** Students will be able to…
To interpret the structure of expressions To write expressions in equivalent forms to solve problems. Perform arithmetic operations on polynomials. Create equations that describe numbers or relationships. Solve equations and inequalities in one variable. Solve systems of equations.	Build on their knowledge from Unit 2, where they extended the laws of exponents to rational exponents. Apply this new understanding of numbers and strengthen their ability to see structure in and create quadratic and exponential expressions. Create and solve equations, inequalities, and systems of equations involving quadratic expressions and determine the values of the function it defines. Understand that polynomials form a system analogous to the integers, they choose and produce equivalent forms of an expression
Assessment Evidence (Stage 2)	
Performance Task Description	

• **Goal** • **Role** • **Audience** • **Situation** • **Product/ Performance** • **Standards**	Engage NY 4 Polynomial and Quadratic Expressions, Equations, and Functions Topic A Quadratic Expressions, Equations, Functions, and Their Connection to Rectangles. **Focus Standards:** A-SSE.A.1 Interpret expressions that represent a quantity in terms of its context.★ a. Interpret parts of an expression, such as terms, factors, and coefficients. b. Interpret complicated expressions by viewing one or more of their parts as a single entity. For example, interpret $(1 + rr)^t$ as the product of $P$and a factor not depending on P. A-SSE.A.2 Use the structure of an expression to identify ways to rewrite it. For example, see $x^4 - y^4$ as $(x^2)^2 - (y^2)^2$, thus recognizing it as a difference of squares that can be factored as $(x^2 - y^2)(x^2 + y^2)$. A-SSE.B.3a Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.★ a. Factor a quadratic expression to reveal the zeros of the function it defines. A-APR.A.1 Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials. A-CED.A.1 Create equations and inequalities in one variable and use them to solve problems. Include equations arising from linear and quadratic functions, and simple rational and exponential functions. ★ A-CED.A.2 Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales. A-REI.B.4b Solve quadratic equations in one variable. b. Solve quadratic equations by inspection (e.g., for $x^2 = 49$), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as $a \pm b$ for real numbers a and b. A-REI.D.11 Explain why the xx-coordinates of the points where the graphs of the equations $y = f(x)$ and $y = g(x)$ intersect are the solutions of the equation $f(x) = g(x)$; find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where (x) and/or (xx) are linear, polynomial, rational, absolute value, exponential, and logarithmic functions.★ F-IF.B.4 For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity. ★ F-IF.B.5 Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. For example, if the function $h(nn)$ gives the number of

Other Evidence
Learning Plan (Stage 3)

• Where are your students headed? Where have they been? How will you make sure the students know where they are going? • How will you hook students at the beginning of the unit? • What events will help students experience and explore the big idea and questions in the unit? How will you equip them with needed skills and knowledge? • How will you cause students to reflect and rethink? How will you guide them in rehearsing, revising, and refining their work? • How will you help students to exhibit and self-evaluate their growing skills, knowledge, and understanding throughout the unit? • How will you tailor and otherwise personalize the learning plan to optimize the engagement and effectiveness of ALL students, without compromising the goals of the unit? • How will you organize and sequence the learning activities to optimize the engagement and achievement of ALL students?	Deep conceptual understanding of operations with polynomials is the focus of this topic. The emphasis is on using the properties of operations for multiplying and factoring quadratic trinomials, including the connections to numerical operations and rectangular geometry, rather than using common procedural gimmicks such as FOIL. In Topic A, students begin by using the distributive property to multiply monomials by polynomials. They relate binomial expressions to the side lengths of rectangles and find area by multiplying binomials, including those whose expanded form is the difference of squares and perfect squares. They analyze, interpret, and use the structure of polynomial expressions to factor, with the understanding that factoring is the reverse process of multiplication. There are two exploration lessons in Topic A. The first is Lesson 6, in which students explore all aspects of solving quadratic equations, including using the zero product property. The second is Lesson 8, where students explore the unique symmetric qualities of quadratic graphs. Both explorations are revisited and extended throughout this topic and the module. In Lesson 3, students encounter quadratic expressions for which extracting the GCF is impossible (the leading coefficient, a, is not 1 and is not a common factor of the terms). They discover the importance of the product of the leading coefficient and the constant (a) and become aware of its use when factoring expressions such as $6x^2 + 5x - 6$. In Lesson 4, students explore other factoring strategies strongly associated with the area model, such as using the area method or a table to determine the product-sum combinations. In Lesson 5, students discover the zero product property and solve for one variable by setting factored expressions equal to zero. In Lesson 6, they decontextualize word problems to create equations and inequalities that model authentic scenarios addressing area and perimeter. Finally, students build on their prior experiences with linear and exponential functions and their graphs to include interpretation of quadratic functions and their graphs. Students explore and identify key features of quadratic functions and calculate and interpret the average rate of change from the graph of a function. Key features include x-intercepts (zeros of the function), y-intercepts, the vertex (minimum or maximum values of the function), end behavior, and intervals where the function is increasing or decreasing. It is important for students to use these features to understand how functions behave and to interpret a function in terms of its context. A focus of this topic is to develop a deep understanding of the symmetric nature of a quadratic function. Students use factoring to reveal its zeros and then use these values and their understanding of quadratic function symmetry to determine the axis of symmetry and the coordinates of the vertex. Often, students are asked to use $x = \frac{-b}{2a}$ as an efficient way of finding the axis of symmetry or the vertex. (Note: Students learn to use this formula without understanding that this is a generalization for the average of the domain values for the x-intercepts.) Only after students develop an

Title of Unit	**Descriptive Statistics**	Grade Level	12th grade SAT prep
Curriculum Area	Mathematics	**Time Frame**	4 weeks
Developed By			
Identify Desired Results (Stage 1)			
Content Standards			

Statistics and Probability - Interpreting Categorical and Quantitative Data
S.ID.1 Represent data with plots on the real number line (dot plots, histograms, and box plots)
. S.ID.2 Use statistics appropriate to the shape of the data distribution to compare center (median, mean) and spread (interquartile range, standard deviation) of two or more different data sets.
S.ID.3 Interpret differences in shape, center, and spread in the context of the data sets, accounting for possible effects of extreme data points (outliers)
S.ID.4 Use the mean and standard deviation of a data set to fit it to a normal distribution and to estimate population percentages
Statistics and Probability - Interpreting Categorical and Quantitative Data
S.ID.5 Summarize categorical data for two categories in two-way frequency tables. Interpret relative frequencies in the context of the data (including joint, marginal, and conditional relative frequencies). Recognize possible associations and trends in the data.
S.ID.6 Represent data on two quantitative variables on a scatter plot, and describe how the variables are related. a. Fit a function to the data; use functions fitted to data to solve problems in the context of the data. Use given functions or choose a function suggested by the context. Emphasize linear and exponential models. b. Informally assess the fit of a function by plotting and analyzing residuals. c. Fit a linear function for a scatter plot that suggests a linear association.
S.ID.7 Interpret the slope (rate of change) and the intercept (constant term) of a linear model in the context of the data.
S.ID.8 Compute (using technology) and interpret the correlation coefficient of a linear fit.
S.ID.9 Distinguish between correlation and causation.

Understandings	**Essential Questions**	
Overarching Understanding	**Overarching**	**Topical**
• A linear function can be used to model the relationship between two numerical variables. • The strength of a relationship and appropriateness of the model used can be determined by analyzing residuals. • A statistical relationship, such as correlation coefficient, is not necessarily the same as a cause and-effect relationship. • The correlation coefficient will be understood and the focus will be on the computation and interpretation of the correlation coefficient as a measure of how well the data fit the relationship. • A deeper look at bivariate data can be taken to describe categorical associations and how to fit models to quantitative data	How would you analyze bivariate data using your knowledge of proportions? How would you describe categorical variables? How would you use your knowledge of functions to fit models to quantitative data? How would you interpret the parameters of a linear model in the context of data that it represents? How can you compute correlation coefficients using	How do you interpret data on a graph? How do you distinguish between a population and a sample? How do you design an experiment? How do you conduct a probability experiment? What is conditional probability? How do you determine if 2 events are mutually exclusive?
Related Misconceptions		

Common Misconceptions: S.CP.6-7 Students may believe: That the probability of A or B is always the sum of the two events individually. That the probability of A and B is the product of the two events individually, not realizing that one of the probabilities may be conditional.	coefficients using technology and interpret the value of the coefficient? How do analysis of bivariate data and knowledge of proportions intersect with each other?	
Knowledge Students will know…	**Skills** Students will be able to…	
To display numerical data and summarize it using measures of center and variability. By the end of middle school they were creating scatterplots and recognizing linear trends in data. To build upon that prior experience, providing students with more formal means of assessing how a model fits data. To use regression techniques to describe approximately linear relationships between quantities. To use graphical representations and knowledge of the context to make judgments about the appropriateness of linear models. With linear models, they look at residuals to analyze the goodness of fit.	Summarize, represent, and interpret data on a single count or measurement variable. *In grades 6 – 8, students describe center and spread in a data distribution. Here they choose a summary statistic appropriate to the characteristics of the data distribution, such as the shape of the distribution or the existence of extreme data points* Summarize, represent, and interpret data on two categorical and quantitative variables. *Students take a more sophisticated look at using a linear function to model the relationship between two numerical variables. In addition to fitting a line to data, students assess how well the model fits by analyzing residuals. S.ID.6b should be focused on linear models, but may be used to preview quadratic functions in Unit 4 of this course* Interpret linear models. *Build on students' work with linear relationships in eighth grade and introduce the correlation coefficient. The focus here is on the computation and interpretation of the correlation coefficient as a measure of how well the data fit the relationship. The important distinction between a statistical* *relationship and a cause-and-effect relationship arises in S.ID.9*	
Assessment Evidence (Stage 2)		
Performance Task Description		

• **Goal** • **Role** • **Audience** • **Situation** • **Product/ Performance** • **Standards**	Engage NY Module 2 Descriptive Statistics **Topic A** **Shapes and Centers of Distributions** **Focus Standards:** S-ID.A.1 Represent data with plots on the real number line (dot plots, histograms, and box plots).★ S-ID.A.2 Use statistics appropriate to the shape of the data distribution to compare center (median, mean) and spread (interquartile range, standard deviation) of two or more different data sets.★ S-ID.A.3 Interpret differences in shape, center, and spread in the context of the data sets, accounting for possible effects of extreme data points (outliers).★ Instructional Days: 3 Lesson 1: Distributions and Their Shapes (P)1 Lesson 2: Describing the Center of a Distribution (E) Lesson 3: Estimating Centers and Interpreting the Mean as a Balance Point (P) **Topic B** **Describing Variability and Comparing Distributions** **Focus Standards:** S-ID.A.1 Represent data with plots on the real number line (dot plots, histograms, and box plots).★ S-ID.A.2 Use statistics appropriate to the shape of the data distribution to compare center (median, mean) and spread (interquartile range, standard deviation) of two or more different data sets.★ S-ID.A.3 Interpret differences in shape, center, and spread in the context of the data sets, accounting for possible effects of extreme data points (outliers).★ Instructional Days: 5 Lesson 4: Summarizing Deviations from the Mean (P)1 Lesson 5: Measuring Variability for Symmetrical Distributions (P) Lesson 6: Interpreting the Standard Deviation (E) Lesson 7: Measuring Variability for Skewed Distributions (Interquartile Range) (E) Lesson 8: Comparing Distributions (E) **Topic C** **Categorical Data on Two Variables** Focus Standards: S-ID.B.5 Summarize categorical data for two categories in two-way frequency tables. Interpret relative frequencies in the context of the data (including joint, marginal, and conditional relative frequencies). Recognize possible associations and trends in the data.★ S-ID.C.9 Distinguish between correlation and causation.★ Instructional Days: 3 Lesson 9: Summarizing Bivariate Categorical Data (E)1 Lesson 10: Summarizing Bivariate Categorical Data with Relative Frequencies (E) Lesson 11: Conditional Relative Frequencies and Association (E)
Other Evidence	

Learning Plan (Stage 3)	
Where are your students headed? Where have they been? How will you make sure the students know where they are going? **How will you hook students at the beginning of the unit?** **What events will help students experience and explore the big idea and questions in the unit? How will you equip them with needed skills and knowledge?** **How will you cause students to reflect and rethink? How will you guide them in rehearsing, revising, and refining their work?** **How will you help students to exhibit and self-evaluate their growing skills, knowledge, and understanding throughout the unit?** **How will you tailor and otherwise personalize the learning plan to optimize the engagement and effectiveness of ALL students, without compromising the goals of the unit?** **How will you organize and sequence the learning activities to optimize the engagement and achievement of ALL students?**	In Topic A, students observe and describe data distributions. They reconnect with their earlier study of distributions in Grade 6 by calculating measures of center and describing overall patterns or shapes. Students deepen their understanding of data distributions, recognizing that the value of the mean and median are different for skewed distributions and similar for symmetrical distributions. Students select a measure of center based on the distribution shape to appropriately describe a typical value for the data distribution. Topic A moves from the general descriptions used in Grade 6 to more specific descriptions of the shape and the center of a data distribution. In Topic B, students reconnect with methods for describing variability that they first used in Grade 6. Topic B deepens students' understanding of measures of variability by connecting a measure of the center of a data distribution to an appropriate measure of variability. The mean is used as a measure of center when the distribution is more symmetrical. Students calculate and interpret the mean absolute deviation and the standard deviation to describe variability for data distributions that are approximately symmetric. The median is used as a measure of center for distributions that are more skewed, and students interpret the interquartile range as a measure of variability for data distributions that are not symmetric. Students match histograms to box plots for various distributions based on an understanding of center and variability. Students describe data distributions in terms of shape, a measure of center, and a measure of variability from the center. In Topic C, students reconnect with previous work in Grade 8 involving categorical data. Students use a two way frequency table to organize data on two categorical variables. Students calculate the conditional relative frequencies from the frequency table. They explore a possible association between two categorical variables using differences in conditional relative frequencies. Students also come to understand the distinction between association of two categorical variables and a causal relationship between two variables. This provides a foundation for work on sampling and inference in later grades

Title of Unit	Trigonometry	Grade Level	12th Sat Prep
Curriculum Area	Mathematics	Time Frame	SAT prep
Developed By			

Identify Desired Results (Stage 1)
Content Standards
G.GPE.6 : Find the point on a directed line segment between two given points that partitions the segment in a given ratio **G.GPE.7:** Use coordinates to compute perimeters of polygons and areas of triangles and rectangles, e.g., using the distance formula.* **G.SRT.4:** Prove theorems about triangles. *Theorems include: a line parallel to one side of a triangle divides the other two proportionally, and conversely; the Pythagorean Theorem proved using triangle similarity.* **G.SRT.5:** Use congruence and similarity criteria for triangles to solve problems and to prove relationships in geometric figures. **G.SRT.9 :** Derive the formula *A* = 1/2 *ab* sin(C) for the area of a triangle by drawing an auxiliary line from a vertex perpendicular to the opposite side. **G.SRT.8 :** Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems. **G.SRT.10:** Prove the Laws of Sines and Cosines and use them to solve problems. **G.SRT.11:** Understand and apply the Law of Sines and the Law of Cosines to find unknown measurements in right and non-right triangles (e.g., surveying problems, resultant forces

Understandings	Essential Questions	
Overarching Understanding	Overarching	Topical

<table>
<tr>
<td>•Express a geometric relationship algebraically (e.g. the Pythagorean Theorem) to new situations such as deriving equation of a circle using the distance formula or deriving the equation of a parabola in terms of focus and directrix.
•Right triangle and triangle similarity can be applied to geometric and algebraic theorems to find coordinates of a point on a line given proportion of segments on the line.
• Justify algebraically the relationships between slopes of parallel and perpendicular lines as they can be established through proof.
• The algebraic representation of a geometric problem can be used to prove theorems in a coordinate plane
•Understand trigonometric ratios as the relationships between sides and angles in right triangles.
• Understand the concept of complementary angles through sine and cosine.
• Trigonometric ratios can be derived for special right triangles (30-60-90 and 45-45-90).
• Real world problems can be solved using right triangles, trigonometric ratios and the Pythagorean theorem.</td>
<td rowspan="3">• Given coordinate plane information, can we prove (or disprove) geometric relationships (e.g. given the vertices, disprove the assertion that ABCD is a rhombus; or that a given point lies on a circle)?
• What is always true about the slopes of perpendicular (or, parallel) lines, and how can a proof be written to exemplify this?</td>
<td rowspan="3">What is right angle similarity?
How is law of sines and cosines used in real life application?
What is trigonometric real life application?</td>
</tr>
<tr>
<td>Related Misconceptions</td>
</tr>
<tr>
<td>G.SRT.6-8
Some students believe that right triangles must be oriented a particular way.
Some students do not realize that opposite and adjacent sides need to be identified with reference to a particular acute angle in a right triangle.
Some students believe that the trigonometric ratios defined in this cluster apply to all triangles, but they are only defined for acute angles in right triangles.</td>
</tr>
<tr>
<td>Knowledge
Students will know…</td>
<td colspan="2">Skills
Students will be able to…</td>
</tr>
<tr>
<td>1. To investigate triangles and decide when they are similar; with this newfound knowledge and their prior understanding of proportional relationships, they define trigonometric ratios and solve problems using right triangles.
2. To explore right triangle trigonometry, and circles and parabolas. Throughout the course, Mathematical Practice 3, “Construct viable arguments and critique the reasoning of others,” plays a predominant role.

3. To advance their knowledge of right triangle trigonometry by applying trigonometric ratios in non-right triangles</td>
<td colspan="2">Use coordinates to prove simple geometric theorems algebraically.

Define trigonometric ratios and solve problems involving right triangles.</td>
</tr>
<tr>
<td colspan="3">Assessment Evidence (Stage 2)</td>
</tr>
</table>

Performance Task Description

• **Goal** • **Role** • **Audience** • **Situation** • **Product/ Performance** • **Standards**	"Solving Geometry Problems: Floodlights" – Mathematics Assessment Project This lesson unit is intended to help you assess how well students are able to identify and use geometrical knowledge to solve a problem. In particular, this unit aims to identify and help students who have difficulty in: •Making a mathematical model of a geometrical situation. •Drawing diagrams to help with solving a problem. •Identifying similar triangles and using their properties to solve problems. •Tracking and reviewing strategic decisions when problem-solving. http://map.mathshell.org/materials/lessons.php?taskid=429#task429 G.SRT.4 "Proofs of the Pythagorean Theorem" – Mathematics Assessment Project This lesson unit is intended to help you assess how well students are able to produce and evaluate geometrical proofs. In particular, this unit is intended to help you identify and assist students who have difficulties in: •Interpreting diagrams. •Identifying mathematical knowledge relevant to an argument. •Linking visual and algebraic representations. •Producing and evaluating mathematical arguments. http://map.mathshell.org/materials/lessons.php?taskid=419#task419 Illustrative Mathematics: "Joining two midpoints of sides of a triangle" http://www.illustrativemathematics.org/illustrations/1095 Illustrative Mathematics: "Pythagorean Theorem" http://www.illustrativemathematics.org/illustrations/1568 "Analyzing Congruence Proofs" – Mathematics Assessment Project This lesson unit is intended to help you assess how well students are able to: •Work with concepts of congruency and similarity, including identifying corresponding sides and corresponding angles within and between triangles. •Identify and understand the significance of a counter-example. •Prove, and evaluate proofs in a geometric context.. http://map.mathshell.org/materials/lessons.php?taskid=452#task452 Illustrative Mathematics: "Bank Shot" http://www.illustrativemathematics.org/illustrations/651 Illustrative Mathematics: "Extensions, Bisections and Dissections in a Rectangle" http://www.illustrativemathematics.org/illustrations/1009 Illustrative Mathematics: "Folding a square into thirds" http://www.illustrativemathematics.org/illustrations/1572 Illustrative Mathematics: "Tangent Line to Two Circles" http://www.illustrativemathematics.org/illustrations/916 Illustrative Mathematics: "Congruence of parallelograms" http://www.illustrativemathematics.org/illustrations/1517 **Tools/Resources:** **G.SRT.6-8** "Geometry Problems: Circles and Triangles" – Mathematics Assessment Project http://map.mathshell.org/materials/lessons.php?taskid=222#task222 Illustrative Mathematics: "Finding the Area of an Equilateral Triangle" http://www.illustrativemathematics.org/illustrations/1322 Illustrative Mathematics: "Mt. Whitney to Death Valley" http://www.illustrativemathematics.org/illustrations/1316

Other Evidence
Learning Plan (Stage 3)

• **Where are your students headed? Where have they been? How will you make sure the students know where they are going?** • **How will you hook students at the beginning of the unit?** • **What events will help students experience and explore the big idea and questions in the unit? How will you equip them with needed skills and knowledge?** • **How will you cause students to reflect and rethink? How will you guide them in rehearsing, revising, and refining their work?** • **How will you help students to exhibit and self-evaluate their growing skills, knowledge, and understanding throughout the unit?** • **How will you tailor and otherwise personalize the learning plan to optimize the engagement and effectiveness of ALL students, without compromising the goals of the unit?** • **How will you organize and sequence the learning activities to optimize the engagement and achievement of ALL students?**	**Instructional Strategies: G.SRT.4-5** Review triangle congruence criteria and similarity criteria, if it has already been established. Review the angle sum theorem for triangles, the alternate interior angle theorem and its converse, and properties of parallelograms. Visualize it using dynamic geometry software. Using SAS and the alternate interior angle theorem, prove that a line segment joining midpoints of two sides of a triangle is parallel to and half the length of the third side. Apply this theorem to a line segment that cuts two sides of a triangle proportionally. Generalize this theorem to prove that the figure formed by joining consecutive midpoints of sides of an arbitrary quadrilateral is a parallelogram. (This result is known as the Midpoint Quadrilateral Theorem or Varignon's Theorem.) Use cardboard cutouts to illustrate that the altitude to the hypotenuse divides a right triangle into two triangles that are similar to the original triangle. Then use AA to prove this theorem. Then, use this result to establish the Pythagorean relationship among the sides of a right triangle ($a2 + b2 = c2$) and thus obtain an algebraic proof of the Pythagorean Theorem. Prove that the altitude to the hypotenuse of a right triangle is the geometric mean of the two segments into which its foot divides the hypotenuse. Prove the converse of the Pythagorean Theorem, using the theorem itself as one step in the proof. Some students might engage in an exploration of Pythagorean Triples (e.g., 3-4-5, 5-12-13, etc.), which provides an algebraic extension and an opportunity to explore patterns. **Explanations and Examples: G.SRT.4** Use AA, SAS, SSS similarity theorems to prove triangles are similar. Use triangle similarity to prove other theorems about triangles o Prove a line parallel to one side of a triangle divides the other two proportionally, and it's converse o Prove the Pythagorean Theorem using triangle similarity. Students may use geometric simulation software to model transformations and demonstrate a sequence of transformations to show congruence or similarity of figures. **Examples:** · Prove that if two triangles are similar, then the ratio of corresponding attitudes is equal to the ratio of corresponding sides. How does the Pythagorean Theorem support the case for triangle similarity? View the video below and create a visual proving the Pythagorean Theorem using similarity. http://www.youtube.com/watch?v=LrS5_l-gk94 To prove the Pythagorean Theorem using triangle similarity: We can cut a right triangle into two parts by dropping a perpendicular onto the hypotenuse. Since these triangles and the original one have the same angles, all three are similar. 

Title of Unit	Circles and Expressing Geometric Properties through Equations	**Grade Level**	12th grade SAT Prep
Curriculum Area	Mathematics	**Time Frame**	4 weeks
Developed By	Munira Jamali		

Identify Desired Results (Stage 1)

Content Standards

G.C.2. Identify and describe relationships among inscribed angles, radii, and chords. Include the relationship between central, inscribed, and circumscribed angles; inscribed angles on a diameter are right angles; the radius of a circle is perpendicular to the tangent where the radius intersects the circle.
G.C.5 Derive using similarity the fact that the length of the arc intercepted by an angle is proportional to the radius, and define the radian measure of the angle as the constant of proportionality; derive the formula for the area of a sector. Convert between degrees and radians. CA
G.MG.1 Use geometric shapes, their measures, and their properties to describe objects (e.g., modeling a tree trunk or a human torso as a cylinder).*
G.GPE.4 Use coordinates to prove simple geometric theorems algebraically. For example, prove or disprove that a figure defined by four given points in the coordinate plane is a rectangle; prove or disprove that the point (1, 3) lies on the circle centered at the origin and containing the point (0, 2)
G.GPE.5. Prove the slope criteria for parallel and perpendicular lines and use them to solve geometric problems (e.g., find the equation of a line parallel or perpendicular to a given line that passes through a given point)
G.GC.3 Construct the inscribed and circumscribed circles of a triangle, and prove properties of angles for a quadrilateral inscribed in a circle
G.GPE.7 Use coordinates to compute perimeters of polygons and areas of triangles and rectangles, e.g., using the distance formula
G.GPE.1 Derive the equation of a circle of given center and radius using the Pythagorean Theorem; complete the square to find the center and radius of a circle given by an equation.

Understandings	**Essential Questions**	
Overarching Understanding	**Overarching**	**Topical**
The algebraic representation of a geometric problem can be used to prove theorems in a coordinate plane. • The concept of similarity as it relates to circles can be extended with proof. • Relationships between angles, radii and chords will be investigated. • Similarities will be applied to derive an arc length and a sector area •Justify algebraically the relationships between slopes of parallel and perpendicular lines as they can be established through proof.	How might we use "constant of proportionality" to define radian measure? • How can we write the equation for a circle or parabola? • How can algebraic representation of a geometric problem be used to prove theorems in coordinate plane? • How can the relationships between angles, radii, and cords be	What is radian measure? How are arc length and area of circle determined by proportionality rule? What is criteria for two line to be parallel or perpendicular What is equation of circle

Related Misconceptions	angles, radii, and cords be investigated? • What is always true about the slopes of perpendicular (or, parallel) lines, and how can a	
Sectors and segments are often used interchangeably in everyday conversation. Care should be taken to distinguish these two geometric concepts. The formulas for converting radians to degrees and vice versa are easily confused. Knowing that the degree measure of given angle is always a number larger than the radian measure can help students use the correct unit. **Common Misconceptions: G.GPE.1-2** Because new vocabulary is being introduced in this cluster, remembering the names of the conic sections can be problematic for some students. The Euclidean distance formula involves squared, subscripted variables whose differences are added. The notation and multiplicity of steps can be a serious stumbling block for some students. The method of completing the square is a multi-step process that takes time to assimilate. A geometric demonstration of completing the square can be helpful in promoting conceptual understanding. **Common Misconceptions: G.GPE.4-7** Students may claim that a vertical line has infinite slopes. This suggests that infinity is a number. Since applying the slope formula to a vertical line leads to division by zero, we say that the slope of a vertical line is undefined. Also, the slope of a horizontal line is 0. Students often say that the slope of vertical and/or horizontal lines is “no slope,” which is incorrect		
Knowledge Students will know…	**Skills** Students will be able to…	

To investigate circles and prove theorems about them. Connecting to their prior experience with the coordinate plane, they prove geometric theorems using coordinates and describe shapes with equations.	Understand and apply theorems about circles. Find arc lengths and areas of sectors of circles. Translate between the geometric description and the equation for a conic section
Assessment Evidence (Stage 2)	
Performance Task Description	
• **Goal** • **Role** • **Audience** • **Situation** • **Product/ Performance** • **Standards**	Illustrative Mathematics: "Right triangles inscribed in circles I" http://www.illustrativemathematics.org/illustrations/1091 Illustrative Mathematics: "Right triangles inscribed in circles II" http://www.illustrativemathematics.org/illustrations/1093 llustrative Mathematics: "Tangent Lines and the Radius of a Circle" http://www.illustrativemathematics.org/illustrations/963 Illustrative Mathematics: "Neglecting the Curvature of the Earth" http://www.illustrativemathematics.org/illustrations/1345 "Equations of Circles 1" – Mathematics Assessment Project This lesson unit is intended to help you assess how well students are able to: •Use the Pythagorean theorem to derive the equation of a circle. •Translate between the geometric features of circles and their equations. http://map.mathshell.org/materials/lessons.php?taskid=406#task406 illustrative Mathematics: "Slopes and Circles" http://www.illustrativemathematics.org/illustrations/479 Illustrative Mathematics: "Explaining the equation for a circle http://www.illustrativemathematics.org/illustrations/1425
Other Evidence	
Learning Plan (Stage 3)	

- Where are your students headed? Where have they been? How will you make sure the students know where they are going?
- How will you hook students at the beginning of the unit?
- What events will help students experience and explore the big idea and questions in the unit? How will you equip them with needed skills and knowledge?
- How will you cause students to reflect and rethink? How will you guide them in rehearsing, revising, and refining their work?
- How will you help students to exhibit and self-evaluate their growing skills, knowledge, and understanding throughout the unit?
- How will you tailor and otherwise personalize the learning plan to optimize the engagement and effectiveness of ALL students, without compromising the goals of the unit?
- How will you organize and sequence the learning activities to optimize the engagement and achievement of ALL students?

Explanations and Examples: G.C.2

Identify central angles, inscribed angles, circumscribed angles, diameters, radii, chords, and tangents.

Describe the relationship between a central angle and the arc it intercepts.

Describe the relationship between an inscribed angle and the arc it intercepts.

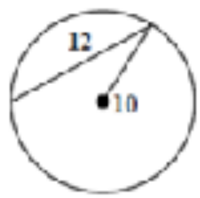
Describe the relationship between a circumscribed angle and the arcs it intercepts.

Recognize that an inscribed angle whose sides intersect the endpoints of the diameter of a circle is a right angle.

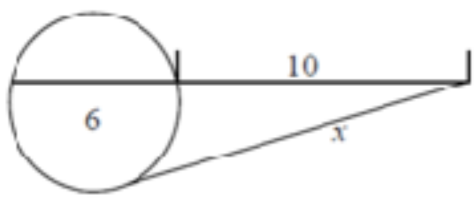
Recognize that the radius of a circle is perpendicular to the tangent where the radius intersects the circle.

Examples:

Given the circle below with radius of 10 and chord length of 12, find the distance from the chord to the center of the circle.

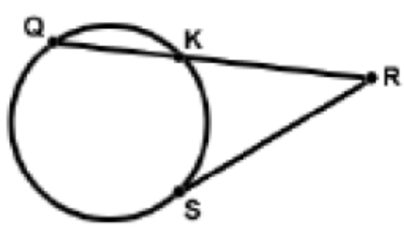


Find the unknown length in the picture below.



Solution:

The theorem for a secant segment and a tangent segment that share an endpoint not on the circle states that for the picture below secant segment QR and the tangent segment SR share and endpoint R, not on the circle. Then the length of SR squared is equal to the product of the lengths of QR and KR .



So for the example above

$$x^2 = 16 \bullet 10$$
$$x^2 = 160$$
$$x = \sqrt{160} = 4\sqrt{10} \approx 12.6$$